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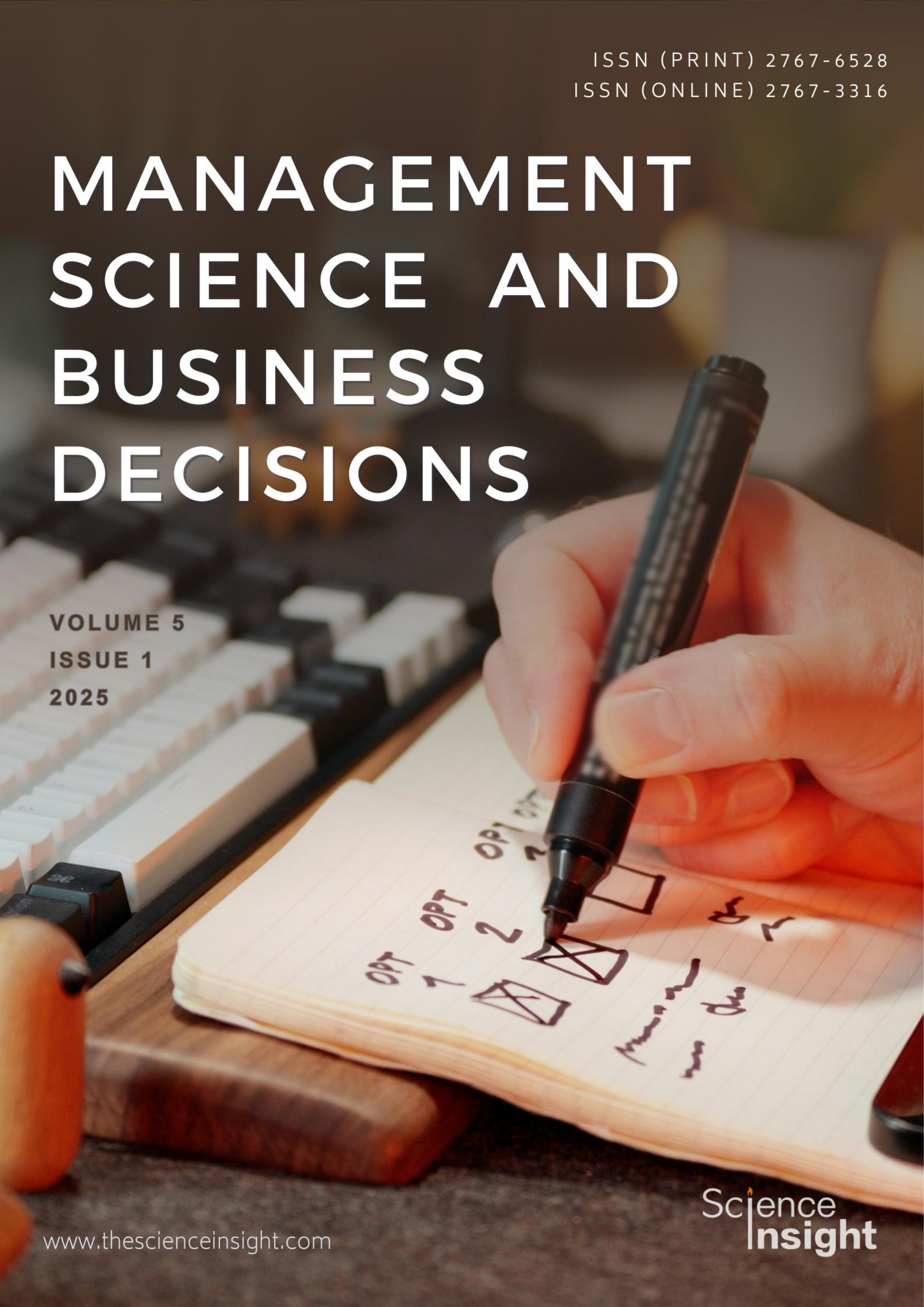

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Analytical Ordinal Priority Approach

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Abstract: The study proposes an analytical (closed-form) solution to the Ordinal Priority Approach (OPA) in multiple attribute decision-making. The proposed Analytical Ordinal Priority Approach (AOPA) can calculate the weights of alternatives, criteria and experts, without linear programming. The application of the AOPA is demonstrated through an example run on Microsoft Excel. The results are consistent with those of the classical OPA. The findings are important for those who seek convenience and may wish to execute the OPA on commonly used spreadsheets without the need for programming languages.

Keywords: Ordinal Priority Approach; analytical; closed-form; multiple criteria decision analysis

1. Introduction

Multiple criteria decision analysis (MCDA) is an important part of operations research and decision theory with applications in numerous disciplines. It provides a structured framework for analysing decision-making problems characterized by complex multiple objectives (Ananda & Herath, 2009). Even though the MCDA problems are diverse they share some common characteristics, e.g., multiple criteria (objectives or attributes), conflict among criteria, incommensurable units and design or selection (Hwang & Yoon, 1991). Most MCDA methods are designed to rank alternatives against conflicting attributes, such as the Grey Relational Analysis (GRA), Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), among others. Most MCDA methods need third-party techniques for estimating the weights of the attributes such as the Rank Order Centroid method (Barron & Barrett, 1996), the Rank Reciprocal method (Stillwell *et al.*, 1981), the Entropy method (Mukhametzyanov, 2021), among others.

When a decision-making problem involves inputs from multiple experts such problems are called multiple criteria group decision-making (MCGDM), or group decision-making (GDM). If one look at numerous literatures on MCGDM, one finds that, despite the important role expert opinions play in these problems, experts are rarely weighed. A common strategy for such problems is the aggregation of the expert judgements using arithmetic or geometric means (Saaty & Vargas, 2007). Thus, integrating the expert weighting mechanism in the theory of MCGDM was one of the least explored areas within the continuously growing scholarship on the MCGDM. In 2020, the Ordinal Priority Approach (OPA) was released that solved the major problem of simultaneous estimation of the weights of experts, attributes and alternatives. Thus, an increasing number of studies are recognizing the OPA as a breakthrough methodology in the field (Aouadni *et al.*, 2024; Čačić *et al.*, 2024; Wang *et al.*, 2022) whereas in just a short span of time, this methodology has seen multiple extensions (Debroy *et al.*, 2025; Du *et al.*, 2024) and several applications in different fields (see, e.g., Pitka *et al.*, 2023; Bah & Tulkinov, 2022; Kiptum *et al.*, 2022).

The OPA is a linear programming-based technique and it requires a computer program (e.g., LINGO, Python, MATLAB, Wolfram Mathematica, etc.) to smoothly execute it. Even though the OPA is garnering increasing recognition with each passing day, it has been observed that there is an immediate need for an analytical (closed-form) solution to the OPA so it can be applied readily and conveniently on popular spreadsheets (e.g., Microsoft Excel, Apple Numbers, Google Sheets, WPS Office Spreadsheet, etc.) available in the computers nowadays around the world. Based on the profound experience of developing and applying the OPA and viewing its applications by other scholars in the market, the authors of the current study have amassed rich insights on the functioning of the OPA. Guided by these insights and observations, along with some empirical evidences, the current study proposes analytical solutions to the OPA to solve the MCGDM problems.

The rest of the study is organized as follows: the next section presents the background that led to the development of the proposed analytical methods for estimating the weights of experts, attributes and alternatives. This section is followed by a section where the proposed system of equations, called Analytical Ordinal Priority Approach (AOPA), is presented. In the subsequent section, the proposed technique is applied on a hypothetical example. Lastly, the study is concluded with some important takeaways.

2. Background

In 2020, the Ordinal Priority Approach was published by a team led by Amin Mahmoudi (Ataei *et al.*, 2020). The OPA method determines the individual weights w_{ijk} by maximizing the objective function Z, which incorporates the ranks of alternatives, attributes and experts. These weights are then summed up to obtain the aggregated weights for alternatives, attributes, and experts. The basic information needed to read the OPA model are shown below.

INDEXES:

i	Index of the experts $(1, \dots, p)$
j	Index of preference of the attributes $(1, \dots, n)$
k	Index of the alternatives $(1, \dots, m)$

SETS:

I	Set of experts $\forall i \in$ I
J	Set of attributes $\forall j \in$ J
K	Set of alternatives $\forall k \in$ K

PARAMETERS:

r_i	The rank of expert i
r_j	The rank of attribute j
r_k	The rank of alternative k

VARIABLES:

Z	Objective function
$W_{ijk}^{r_k}$	Weight (importance) of k^{th} alternative based on j^{th} attribute by i^{th} expert at r_k^{th} rank

The following linear programming model represents the classical OPA and is supposed to be solved using a programming language (Mahmoudi & Javed, 2023a),

$$\begin{aligned}
& Max\ Z \\
& s.t: \\
& Z \leq r_i\left(r_j\left(r_k\left(W_{ijk}^{r_k} - W_{ijk}^{r_{k+1}}\right)\right)\right) \qquad \forall\ i,j \text{ and } r_k \\
& Z \leq\ r_i r_j r_m W_{ijk}{}^{r_m} \qquad \forall\ i,j \text{ and } r_k = r_m \\
& \sum_{i=1}^{p}\sum_{j=1}^{n}\sum_{k=1}^{m} w_{ijk} = 1 \\
& W_{ijk} \geq 0 \qquad \forall\ i,j \text{ and } k
\end{aligned} \tag{1}$$

where Z is unrestricted in sign.

After solving Model (1), the experts' weights can be determined by employing Eq. (2).

$$W_i = \sum_{j=1}^{n}\sum_{k=1}^{m} W_{ijk} \qquad \forall\ i \tag{2}$$

To calculate the weights of the attributes, Eq. (3) can be utilized.

$$W_j = \sum_{i=1}^{p}\sum_{k=1}^{m} W_{ijk} \qquad \forall\ j \tag{3}$$

And, the alternatives' weight can be calculated using Eq. (4).

$$W_k = \sum_{i=1}^{p}\sum_{j=1}^{n} W_{ijk} \qquad \forall\ k \tag{4}$$

Mahmoudi and Javed (2023a) extended the OPA through interval mathematics. In their work some important findings were reported related to the theory of the OPA. They illustrated that as the number of objects (e.g., attributes or alternatives) to be ranked increases, the difference in importance between them gets smaller as one moves from top ranked objects to lower ranked objects. It means, the difference between the ranks 1 and 2 is larger than the difference between the ranks 2 and 3, and so on. There are several rank-based methods that exhibit these properties. Further, they stated that,

> "In fact, two of the rank-based methods—rank reciprocal (Stillwell *et al.* 1981) and rank order centroid (Barron and Barrett 1996)—are special cases of the Ordinal Priority Approach on the criterion weighting and alternative weighting dimensions, respectively."

Therefore, "in the OPA, two competing models, traditionally used for estimating the weights of attributes, complement each other" (Javed & Du, 2023). In Proposition 2 and Definition 2 of Mahmoudi and Javed (2023a), they argued that the Rank Reciprocal method is a special case of the OPA for estimating the weights of attributes (criteria), when all experts are equally important or when there is only one expert. Based on this construct later they derived important results.

In their Proposition 1 and Definition 1, they argued that the Rank Order Centroid method is a special case of the OPA for estimating the weights of alternatives, when all experts are equally important or when there is only one expert. Meanwhile, it should be noted that in the classical OPA (Ataei *et al.*, 2020), weight (importance) of k^{th} alternative is not absolute, but is defined relative to j^{th} criterion and i^{th} expert at r^{th} rank. Actually, the first constraint of the OPA model indirectly manifests this construct. If one look at the first inequality of the OPA model,

$$Z \leq r_i \left(r_j \left(r_k \left(W_{ijk}^{r_k} - W_{ijk}^{r_{k+1}} \right) \right) \right) \qquad \forall\, i, j \text{ and } r_k \tag{5}$$

one can observe that the importance of an alternative at a given rank, based on a specific expert and attribute, is tied to the expert's rank (r_i) and the attribute's rank (r_j). The higher the ranks of the expert and attribute, the more significant the difference in weights between consecutive alternative ranks for Z. Similar conclusions can be drawn from the careful reading of Mahmoudi and Javed (2023b). Also, as the core decision variable, $W_{ijk}^{r_k}$, is the weight (importance) of k^{th} alternative based on j^{th} attribute by i^{th} expert at r_k^{th} rank therefore, one can argue that the weight of alternative is the function of the rank of alternatives as well as the rank (and thus, weight) of attribute and rank (and, thus, weight) of expert, i.e., at the r_k^{th} rank,

$$W_k = f(r_k, r_j, r_i), \tag{6}$$

or, more precisely,

$$W_{ijk} = f(r_{ijk}, r_{ij}, r_i), \tag{7}$$

Meanwhile, in another work (Mahmoudi & Javed, 2022), they clearly argued that the qualification of experts is the prerequisite to the qualification of attributes (criteria), which in turns is a prerequisite to the qualification of alternatives. Thus, in the OPA the weights are hierarchically determined, i.e., each object's weight (importance) is influenced by its position relative to other ranked objects. Mahmoudi and Javed (2023a) defined the weight estimated through the OPA as a probability of a given object's priority over the other. Thus, in the OPA, it's common to write "weights (importance)" (Ataei *et al.*, 2020) because the OPA weights are not necessarily the "weights." Depending on a situation, they can denote probabilities (Mahmoudi & Javed, 2023a; Javed & Du, 2023) or something else as well. Based on the authors' understanding of the behaviour of the weights of the OPA (as the number of objects increase), and the properties of the OPA, three axioms and few propositions are advanced in the current study:

AXIOM 1: Weight (Expert) = f(Rank (Expert)).

AXIOM 2: Weight (Attribute) = f(Rank (Attribute), Rank (Expert)).

AXIOM 3: Weight (Alternative) = f(Rank (Alternative), Rank (Attribute), Rank (Expert)).

These three axioms are proven from the discussion that preceded them.

PROPOSITION 1: In the OPA, a "weight" is a unit interval value (or scaled value) that represents the relative behaviour of ranked objects. A "weight" in one case may be conceptualized as a "probability" in another case and an index (or score) of relative importance (or performance) in another case.

It is proven from literature (Mahmoudi & Javed, 2023a; Javed & Du, 2023).

PROPOSITION 2: In the OPA, the weights (or importance) are hierarchically determined i.e., the position of objects (alternative, attribute, and expert) relative to each other matters.

It is proven from Axioms 1 to 3.

3. Analytical Ordinal Priority Approach

The Analytical Ordinal Priority Approach (AOPA) is the analytical equivalent of the classical linear programming-based Ordinal Priority Approach (OPA). Given the data is complete and there is no tie, in this approach the weights of the experts, attributes and alternatives would be calculated using the following definitions.

DEFINITION 1: Expert weights

In a three-dimensional multiple attribute group decision making problem, if r_i is the rank of i^{th} expert and total number of experts are p, then the weight of i^{th} expert will be given as

$$W_i = \frac{\frac{1}{r_i}}{\sum_{i=1}^{p} \frac{1}{r_i}} \tag{8}$$

These weights are absolutely consistent with the weights calculated using the Rank Reciprocal method, if applied on experts.

DEFINITION 2: Attribute weights

In a multiple attribute group decision making problem, if r_i is the rank of i^{th} expert, and r_{ij} is the rank of j^{th} attribute against i^{th} expert whereas, the total number of experts are p and the total number of attributes are n, then the weight of j^{th} attribute will be given as

$$W_j = \frac{u_j}{\sum_{j=1}^{n} u_j} \tag{9}$$

where,

$$u_j = \sum_{i=1}^{p} \left(\frac{1}{r_i r_{ij}} \right) \tag{10}$$

or, simply,

$$W_j = \frac{\sum_{i=1}^{p} \left(\frac{1}{r_i r_{ij}} \right)}{\sum_{j=1}^{n} \left(\sum_{i=1}^{p} \left(\frac{1}{r_i r_{ij}} \right) \right)} \tag{11}$$

The attribute weight estimation method is a direct generalization of the expert weight estimation method. If a problem involves only one expert (or all experts are equally important), the formula of the attribute weight would have a structure similar to that of the expert weight.

DEFINITION 3: Alternative weights

In a multiple attribute group decision making problem, let us assume that r_i is the rank of i^{th} expert, and r_{ij} is the rank of j^{th} attribute against i^{th} expert and r_{ijk} is the rank of k^{th} alternative against j^{th} attribute assigned by i^{th} expert. If the total number of experts are p, the total number of attributes are n, and the total number of alternatives are m, then the weight of k^{th} alternative will be given as,

$$W_k = \frac{v_k}{\sum_{k=1}^{m} v_k} \tag{12}$$

where,

$$v_k = a_{1j1} + a_{2jk} + \cdots + a_{pjk} = \sum_{i=1}^{p} a_{ijk} \tag{13}$$

where,

$$a_{ijk} = \sum_{j=1}^{n} \left(\frac{1}{r_i r_{ij}} \times \sum_{r_{ijk}=k}^{p} \frac{1}{r_{ijk}} \right) \tag{14}$$

It should be noted that a_{ijk} is a very interesting coefficient. On right hand side, the first part $\left(\frac{1}{r_i r_{ij}}\right)$ is inspired by the rank reciprocal operation of the Rank Reciprocal method while the second part $\left(\sum_{r_{ijk}=k}^{p} \frac{1}{r_{ijk}}\right)$ is inspired by the rank aggregation operation of the Rank Order Centroid method. Thus, a_{ijk} represents a novel contribution of the AOPA to the decision theory. In short, the weight of k^{th} alternative will be given as,

$$W_k = \frac{\sum_{i=1}^{p} \left(\sum_{j=1}^{n} \left(\frac{1}{r_i r_{ij}} \times \sum_{r_{ijk}=k}^{p} \frac{1}{r_{ijk}} \right) \right)}{\sum_{k=1}^{m} \left(\sum_{i=1}^{p} \left(\sum_{j=1}^{n} \left(\frac{1}{r_i r_{ij}} \times \sum_{r_{ijk}=k}^{p} \frac{1}{r_{ijk}} \right) \right) \right)} \tag{15}$$

The alternative weight estimation method is a complex generalization of the attribute weight estimation method. When we have one expert (or all experts are equally important), and one attribute (or all attributes are equally important), the formula of the alternative weight would have a structure similar to that of the attribute weight.

Now another exercise can be done, for the sake of convenience of our readers who want to apply the AOPA with further ease. If we assume that

$$\mathrm{g} = \sum_{r_{ijk}=k}^{p} \frac{1}{r_{ijk}} \tag{16}$$

Then a g-score table can be constructed for quick reference. The g-scores represent transformation of the ranks r_{ijk}. The g-score table for up to twenty alternatives is shown in *Table 1*. *Table 1* can be used for any group decision-making problem that involves two to twenty alternatives. For larger problems, it can be extended by using Eq. (16). It should be noted that the sum of each column containing the g-scores equals the number of alternatives. Those users who may need to extend this table, can use this point to double check their calculations.

4. Application

In this section the AOPA and the OPA will be applied on a hypothetical case involving three experts ($p = 3$), four attributes ($n = 4$), and five alternatives ($m = 5$).

4.1 Calculating weights of the experts

It is believed that the first expert (E1) is considered more authoritative than the second expert (E2), who is considered more authoritative than the third expert (E3), i.e.,

E1 > E2 > E3.

Thus, by applying Eq. (8), the results that we got are shown in *Table 2*. For comparative analysis, the OPA weights are also shown in the last column of the table. One can see that the expert weights obtained through the AOPA are exactly same like those obtained through the OPA. The first expert got 54.5% weight, while the second and third experts got 27.3% and 18.2%, respectively.

Table 1. The table of g-scores

m^{-1}	m	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1.000	1	1.500	1.833	2.083	2.283	2.450	2.593	2.718	2.829	2.929	3.020	3.103	3.180	3.252	3.318	3.381	3.440	3.495	3.548	3.598
0.500	2	0.500	0.833	1.083	1.283	1.450	1.593	1.718	1.829	1.929	2.020	2.103	2.180	2.252	2.318	2.381	2.440	2.495	2.548	2.598
0.333	3		0.333	0.583	0.783	0.950	1.093	1.218	1.329	1.429	1.520	1.603	1.680	1.752	1.818	1.881	1.940	1.995	2.048	2.098
0.250	4			0.250	0.450	0.617	0.760	0.885	0.996	1.096	1.187	1.270	1.347	1.418	1.485	1.547	1.606	1.662	1.714	1.764
0.200	5				0.200	0.367	0.510	0.635	0.746	0.846	0.937	1.020	1.097	1.168	1.235	1.297	1.356	1.412	1.464	1.514
0.167	6					0.167	0.310	0.435	0.546	0.646	0.737	0.820	0.897	0.968	1.035	1.097	1.156	1.212	1.264	1.314
0.143	7						0.143	0.268	0.379	0.479	0.570	0.653	0.730	0.802	0.868	0.931	0.990	1.045	1.098	1.148
0.125	8							0.125	0.236	0.336	0.427	0.510	0.587	0.659	0.725	0.788	0.847	0.902	0.955	1.005
0.111	9								0.111	0.211	0.302	0.385	0.462	0.534	0.600	0.663	0.722	0.777	0.830	0.880
0.100	10									0.100	0.191	0.274	0.351	0.423	0.489	0.552	0.611	0.666	0.719	0.769
0.091	11										0.091	0.174	0.251	0.323	0.389	0.452	0.511	0.566	0.619	0.669
0.083	12											0.083	0.160	0.232	0.298	0.361	0.420	0.475	0.528	0.578
0.077	13												0.077	0.148	0.215	0.278	0.336	0.392	0.445	0.495
0.071	14													0.071	0.138	0.201	0.259	0.315	0.368	0.418
0.067	15														0.067	0.129	0.188	0.244	0.296	0.346
0.063	16															0.063	0.121	0.177	0.230	0.280
0.059	17																0.059	0.114	0.167	0.217
0.056	18																	0.056	0.108	0.158
0.053	19																		0.053	0.103
0.050	20																			0.050

Table 2. The estimation of expert weights using Analytical OPA

	Rank	$1/r_i$	W_i (AOPA)	W_i (OPA)
E1	1	1.000	0.545	0.545
E2	2	0.500	0.273	0.273
E3	3	0.333	0.182	0.182

4.2 Calculating weights of the attributes

Each expert ranked the four attributes (C1, C2, C3, C4) differently. For instance, for the first expert, the first attribute is more important than the second attribute, which is more important than the third attribute, which in turns is considered least important.

$$C1 > C2 > C3 > C4.$$

While for the second expert,

$$C4 > C3 > C2 > C1$$

and for the third expert,

$$C4 > C1 > C2 > C3.$$

These ranks are shown in *Table 3*, along with the results obtained through the applications of Eqs. (9) and (10). One can see that the attribute weights obtained through the AOPA are consistent with those obtained through the OPA. It is found that the first attribute is most important with

33.8% weight, while the fourth, third and second attributes got 28.4%, 20.4% and 17.5% weights, respectively. Thus, overall,

$$C1 > C4 > C2 > C3.$$

4.3 Calculating weights of the alternatives

In the end, each expert evaluated each of the five alternatives (A1, A2, A3, A4, A5) against each attribute, and the decision matrix is shown in *Table 4*. Their g-transformations, obtained using Eq. (16) or *Table 1*, are shown in *Table 5*. The results obtained through the application of Eqs. (14), (13) and (12) are shown in *Table 6*. One can see that the alternative weights obtained through the AOPA are consistent with those obtained through the OPA. It is found that the first alternative is most important with 27.1% weight. It is followed by the fourth alternative (19.3%), the third alternative (18.3%), the second alternative (17.9%) and the fifth alternative (17.3%). Thus, overall,

$$A1 > A4 > A3 > A2 > A5.$$

5. Conclusion

The study proposed the analytical (closed-form) form of the Ordinal Priority Approach (OPA), a multiple attribute decision-making methodology. Through an application it has been shown that

Table 3. The estimation of attribute weights using Analytical OPA

		C1	C2	C3	C4
Rank	E1	1	2	3	4
	E2	4	3	2	1
	E3	2	3	4	1
$\frac{1}{r_i r_{ij}}$	E1	1.000	0.500	0.333	0.250
	E2	0.125	0.167	0.250	0.500
	E3	0.167	0.111	0.083	0.333
u_j		1.292	0.778	0.667	1.083
W_j (AOPA)		0.338	0.204	0.175	0.284
W_j (OPA)		0.338	0.204	0.175	0.284

Table 4. The decision matrix containing the ranks r_{ijk}

	E1				E2				E3			
	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
A1	1	2	3	4	5	5	4	1	1	3	4	5
A2	2	3	4	5	1	4	3	3	3	2	3	3
A3	3	4	5	1	2	3	2	2	5	5	2	2
A4	4	5	1	2	3	2	1	5	4	4	1	1
A5	5	1	2	3	4	1	5	4	2	1	5	4

Table 5. The g-scores associated with r_{ijk}

	E1				E2				E3			
	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
A1	2.283	1.283	0.783	0.450	0.200	0.200	0.450	2.283	2.283	0.783	0.450	0.200
A2	1.283	0.783	0.450	0.200	2.283	0.450	0.783	0.783	0.783	1.283	0.783	0.783
A3	0.783	0.450	0.200	2.283	1.283	0.783	1.283	1.283	0.200	0.200	1.283	1.283
A4	0.450	0.200	2.283	1.283	0.783	1.283	2.283	0.200	0.450	0.450	2.283	2.283
A5	0.200	2.283	1.283	0.783	0.450	2.283	0.200	0.450	1.283	2.283	0.200	0.450

Table 6. The table containing a_{ijk}, v_k and W_k

	E1	E2	E3	v_k	W_k (AOPA)	W_k (OPA)
A1	3.299	1.313	0.572	5.183	0.271	0.271
A2	1.875	0.948	0.600	3.422	0.179	0.179
A3	1.646	1.253	0.590	3.490	0.183	0.183
A4	1.632	0.983	1.076	3.691	0.193	0.193
A5	1.965	0.712	0.634	3.311	0.173	0.173

the weights generated by the proposed Analytical Ordinal Priority Approach (AOPA) are consistent with those from the classical OPA. In future, the authors would extend the AOPA to incorporate datasets that include incompleteness and ties.

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Disclosure of Conflict of Interest

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Forecasting the Demand for Human Resources in a Hospital using the Grey Forecasting Model

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Abstract: The current study applies the grey forecasting model GM(1,1) to forecast human resource demand in a private hospital in Hebei, a province of China. With the analysis of its 2020-2024 staffing data, the model predicts a steadily increased number during 2025 to 2027. The forecast accuracy was examined by Mean Absolute Percentage Error. The study concludes with some suggestions for manpower demand management in the hospital.

Keywords: Human resource management; demand forecast; hospital; grey model; grey forecast

1. Introduction

In August 2022, China's National Health Commission issued the 14th Five-Year Plan for the Development of Health Personnel, which clearly states that the overall objective for the period is to enhance service capabilities and optimize the structure of healthcare professionals, improve personnel management systems and mechanisms, and foster a sound environment for talent development (National Health Commission, 2022). This highlights the critical role of medical, nursing, and pharmaceutical professionals in advancing the nation's healthcare system and improving the overall physical well-being of the population.

It has been observed that many previous studies have focused their main efforts on predicting the demand for health technical personnel in large population samples such as provinces and cities, or even countries. For instance, Guan *et al.* (2010), studied the numbers of medical specialists and beds between 2009 and 2020 in Chengdu mathematically using a grey forecasting model and guided by the development plans of local health administration. Liu *et al.* (2011) predicted the medical staff needs in Beijing communities using the indicators of staffing need (ISN). Scheffler and Arnold (2019) projected the shortage of doctors and nurses in Organization for Economic Co-operation and Development (OECD) countries till 2030. Huang *et al.* (2020) performed the demand forecast of health resources in Shanxi traditional Chinese medicine hospitals and annual average growth rate of medical human power. Lv *et al.* (2024) studied medical and health resources in Anhui Province from 2009 to 2020. In general, it is observed that predicting hospital-specific personnel needs, especially in the context of Hebei, is an area that requires further exploration. A very brief overview of the selected literature is shown in *Table 1*. The current study will analyze staffing data from a specific hospital in Hebei from 2020 to 2024 while constructing a grey model GM(1,1) to

Table 1. A brief overview of some studies

Year	Keywords	Study area	Methods	Literature
2010	Health Resources; Development Plan	Chengdu City	GM(1,1)	Guan *et al.* (2010)
2011	Community Health Service	Beijing communities	ISN	Liu *et al.* (2011)
2019	Health Workforce	OECD countries	ARIMA	Scheffler and Arnold (2019)
2020	Health Resources	Shanxi traditional Chinese medicine hospitals	GM(1,1)	Huang *et al.* (2020)
2024	Health Resources	Anhui Province	GM(1,1)	Lv *et al.* (2024)
2025	Hospital	A hospital in Hebei Province	GM(1,1)	The current study
NOTE: ARIMA = Autoregressive Integrated Moving Average Models, ISN = Indicators of Staffing Need				

forecast hospital personnel needs in the next few years, and compares the prediction with actual figures.

The rest of the study is organized as follow. In the second section, data collection In the third section, the model's accuracy is validated using MAPE (Mean Absolute Percentage Error), and the hospital's human resource demand over the next three years is projected for reference. In the last section, the study has been concluded.

2. Research Methodology

2.1 Data

The hospital under study is a private hospital located in Hebei, a province of the People's Republic of China. Among its popular disciplines include orthopedics and proctology. The data used in the current study was responsibly obtained from the human resources department of the hospital and was covering the years 2020 to 2024. The data included total staff numbers, age distributions, and professional title structures. The year 2025 is predicted and then compared with actual data, followed by projecting the demand of active employees in future.

2.2 Grey Model GM(1,1)

The Grey System Theory was first proposed by Professor Deng Julong in 1982. Among its major streams of research includes grey forecasting models. The grey models are characterized by low data requirements, small computational needs, and high accuracy for short- to medium-term forecasts, especially when data distribution is regular (Liu *et al.*, 2010). Given that HR demand in a hospital is influenced by factors such as medical school enrollment, residency training policies, and regional healthcare planning—factors with unclear linear relationships—this study relies solely on historical data for short-term forecasting. The current study used the classical grey forecasting model GM (1,1). WPS Office Spreadsheet tool was used for data analysis. The grey model GM model used in the current study has been adapted from Zhao *et al.* (2024).

3. Research Methodology

3.1 Basic Staffing Situation from 2020 to 2024

The staffing overview is shown in *Table 2*, with age distribution in *Table 3*. Staff transfers and deaths are excluded. From *Table 2*, staff numbers increased from 743 in 2020 to 813 in 2024, reflecting a growth rate of approximately 9.42% and an annual average growth rate of about 2.29%. The proportion of junior titles decreased, while intermediate and senior titles increased. Intermediate titles rose from 26.51% to 39.36%, and senior titles rose from 19.38% to 20.54%.

Significant data fluctuations were observed in 2022. There can be several reasons of this disturbance in data: (1) It was a post-COVID-19 period in China. It is also possible that the retirement and resignation of a small number of personnel have led to a reduction in the overall

Table 2. The distributions of total hospital staff and title

	Total Staff	Junior Titles	Intermediate Titles	Senior Titles
2020	743	402	197	144
2021	788	436	209	143
2022	779	258	353	168
2023	804	393	263	148
2024	813	326	320	167

Table 3. The distribution of staff age

	Age ≤ 23	24–33	34–43	44–53	54–63	≥64
2020	0	266	238	175	63	1
2021	2	271	268	186	61	0
2022	0	236	303	178	61	1
2023	2	225	335	179	63	0
2024	0	206	354	203	50	0

base, and (2), the policy of professional title promotion that supports front-line personnel in the fight against the COVID-19 may have resulted in a significant increase in the number of personnel with intermediate and senior professional titles. Compared to 2021, intermediate titles increased by 144 (70.85%), and senior titles increased by 25 (18.84%). However, in 2023, intermediate titles fell by 90 (-27.81%), and senior titles also decreased by 20 (-14.64%).

As shown in *Table 3*, the ≤23 and ≥64 age groups are statistically insignificant. The remaining four groups center around an average of 40.8, 299.6, 184.2, 59.6, with minimal variation.

3.2 Forecast for 2025–2027

3.2.1 GM(1,1) Model Construction. We build the original sequence $X^{(0)}$= [743, 788, 779, 804, 813] and generate the cumulative sequence $X^{(1)}$:

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i)$$

Then,

$$X^{(1)} = [X^{(1)}(1), X^{(1)}(2), X^{(1)}(3), X^{(1)}(4), X^{(1)}(5)]$$

$$X^{(1)} = [743,1531,2310,3114,3927].$$

Next, the mean adjacent sequence $Z^{(1)} = [Z^{(1)}(2), Z^{(1)}(3), Z^{(1)}(4), Z^{(1)}(5)]$ is derived:

$$Z^{(1)}(k) = \frac{1}{2}\left(X^{(1)}(k) + X^{(1)}(k-1)\right), k = 2,3,4,5$$

We then establish the grey differential equation:

$$X^{(0)}(k) + aZ^{(1)}(k) = b, k = 2,3,4,5$$

The corresponding whitened differential equation is:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b$$

Later, the least squares method is used to estimate a and b:

$$(a,b)^T = (B^T B)^{-1} B^T Y$$

Let B and Y be defined as:

$$B = \begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ -Z^{(1)}(4) & 1 \\ -Z^{(1)}(5) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ X^{(0)}(4) \\ X^{(0)}(5) \end{bmatrix}$$

We find development coefficient $a = -0.013$ and grey action quantity $b = 766.692$, resulting in the time response function:

$$\hat{X}^{(1)}(k+1) = \left[X^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a}, k = 0,1,2,3,4$$

Back-transforming yields the forecasted original data sequence:

$$\hat{X}^{(0)}(k+1) = \hat{X}^{(1)}(k+1) - \hat{X}^{(1)}(k) = (1 - e^{a}) \left[X^{(0)}(1) - \frac{b}{a}\right] e^{-ak}$$

For $k = 6$ (year 2025), the forecast is 821.419, aligning closely with the actual count of 808 reported in April 2025. Later, Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE) or other metrics can be used to quantify the disparity between simulated and actual numbers.

3.2.2 Forecast for 2025–2027. The forecast for next three years is estimated as:

For $k = 6$ (year 2025), $\hat{X}^{(0)}(6) = 821.419$, in the vicinity of 821.

For $k = 7$ (year 2026), $\hat{X}^{(0)}(7) = 831.850$, in the vicinity of 832.

For $k = 8$ (year 2027), $\hat{X}^{(0)}(8) = 842.414$, in the vicinity of 842.

The GM(1,1) predicted a steady increase in staff over the next three years, with an annual average growth rate of approximately 1.27%. Compared to 2020, the 2027 staff count increases by 13.32%, averaging 1.8% annually. See *Figure 1* for the trend.

3.3 Forecast Error Testing

Mean Absolute Percentage Error (MAPE) is an effective measure of forecast accuracy. It uses actual and predicted values to estimate an error index that can be is interpretable in percentage terms. It is the average of the absolute percentage errors between forecasts and actuals (Duan & Nie, 2022), and is given by

$$MAPE(\%) = \frac{1}{n} \sum_{i=1}^{n} \frac{|y_i - \hat{y}_i|}{y_i} \times 100$$

where n is the number of observations, y_i is the actual value, and $\hat{y}_i$ is the forecast. The MAPE values can be interpreted using the scale shown in *Table 4*, adapted from Javed and Cudjoe (2022).

As mentioned above, the actual value of 2020-2024 were trained in grey model GM(1,1) as in-sample data, however, the MAPE value of in-sample data should also be tested to prove the model's accuracy. The authors decided to use the personnel number of 2024 to justify whether this

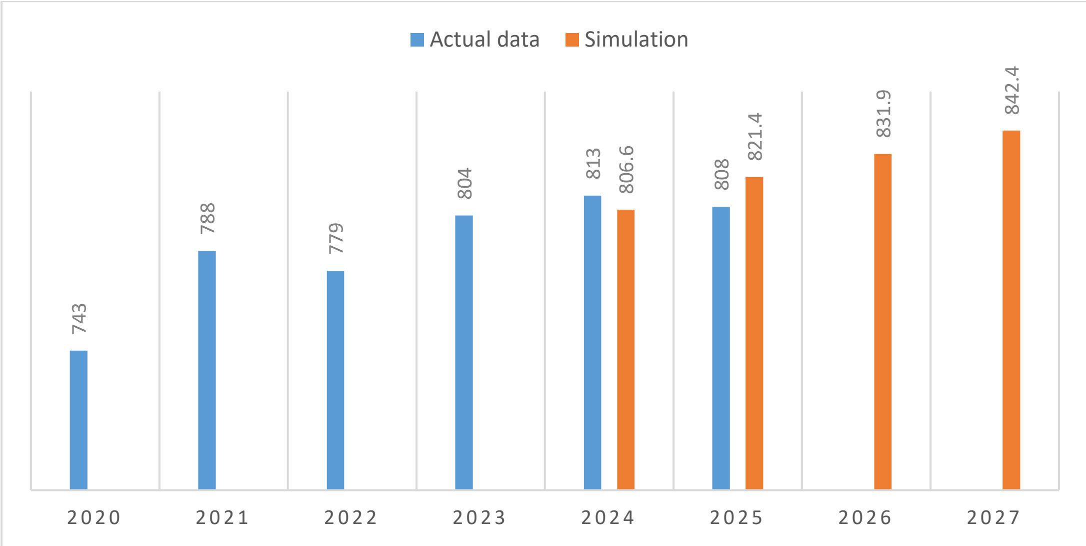

Figure 1. Actual and simulated hospital staff numbers.

Table 4. The MAPE scale

MAPE	Accuracy
<10%	Very good, high fit
10–20%	Good, acceptable error
20–30%	Moderate, large error
>30%	Poor prediction

model will be applied to predict a future human demand. The actual number of 2024 is 813 whereas the forecast is 806.579, in the vicinity of 806. And the MAPE value is 0.79%, according to MAPE scale reported in Table 4, it indicates a strong predictive accuracy. Out-sample data comes to the year of 2025. While the actual value is 808, collected at the end of April of this year, the forecast is 821.419. And, the MAPE value of GM(1,1) model in this case is 1.66%, indicating strong forecast accuracy.

4. Conclusion and recommendation

Medical staff are the backbone of hospital operations and public health progress. Their employment stability and career development directly affect healthcare quality. Currently, challenges include long training cycles (Liu & Wang, 2019) and increasing personnel loss (Chen & Chen, 2024). To align HR supply with demand, organizations should enhance the talent ecosystem on multiple levels:

4.1 Improving Promotion Systems

Establish clear, fair, and transparent promotion standards beyond tenure and academic credentials. Evaluate professional skills, work performance, research achievements, and patient satisfaction. Provide clear career paths, with distinct responsibilities and benefits per rank. Conduct regular internal training, invite industry experts for lectures, and encourage participation in domestic and international conferences and training. Collaborate with universities for continuing education programs to help staff update their knowledge and keep pace with medical advances (Cheng *et al.*, 2009).

4.2 Optimizing Compensation and Benefits

Ensure competitive pay to attract and retain talent. Link performance pay to job quality, workload, and patient satisfaction. Provide front-line staff with risk allowances. Beyond legal benefits, offer perks like commercial insurance, health checks, paid leave, holiday bonuses, discounted meals, and housing subsidies. Staff with children should be support by assisting their children with school admissions. Housing assistance for non-locals should be provided, and exceptional performance in innovation or crisis response should be rewarded with bonuses and honors.

4.3 Building a Positive Culture

Upgrade hospital facilities and equipment for a safer, more comfortable work environment. Optimize workflows and layout to reduce unnecessary burdens. Promote digitalization for faster information sharing and higher efficiency. Foster a collaborative, respectful, and patient-focused culture. Organize team-building events, birthdays, and cultural activities to strengthen cohesion and workplace relationships. Ensure smooth communication with patients to resolve disputes, uphold staff dignity, and create a safe, supportive, and worry-free environment where staff can focus on care.

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