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What is the Ordinal Priority Approach?

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Abstract: Humans, in their most important role as decision-makers, make choices every day. Simple choices don't even require a calculator, but more complicated ones require some calculation. Complicated choices, such as the ones where one has to choose one (or multiple) alternatives from a list of alternatives, whereas each alternative may be better than the other on different conflicting attributes, are not easy to make. Such problems are usually referred to as multiple attribute decision-making (MADM), and one of the easiest and most effective ways to solve such problems is the Ordinal Priority Approach (OPA). In this article, readers will learn: (1) what is the OPA (and some discussion on its background), (2) what distinguishes it from other MADM methods, (3) its scale, and (4) how to apply it. Readers, including those with the most elementary knowledge of mathematics and operations research, would find this article particularly useful as every effort has been made to make content easy to comprehend.

Keywords: Business decision making; multiple criteria decision analysis; ordinal priority approach; ordinal preference relation; LINGO software

1. Introduction

Aristotle, in his work Nicomachean Ethics, defined "preferences" as "rational desires" and, thus, perhaps made one of the earliest attempts to relate rational decision-making to human desires (preferences) (Köksalan *et al.* 2016). Preferences play a very important role in decision-making. Numeric scores (precise/ cardinal numbers) may be useful when there is nothing to compare with, however, when we have something to compare with (a very probable scenario in real-life), ranks (manifestation of preferences) provide a very convenient way to make choices (Agresti, 2010). In real life, we mostly have something to compare with, e.g., the buyer of an electric vehicle might be comparing it (in his mind) with other electric vehicle models or internal combustion engine vehicles before making any purchase decision. Also, it is easier to rank two products based on one's overall experience using it, but it's not easy to assign precise numbers. The statement "your daughter scored 45 marks" is almost meaningless to the student's parent(s); however, where she stands with respect to her classmates is what matters most to the parent(s). In situations involving subjectivity, where human observers (raters) can have different perspectives on the same topic, preferences provide the easiest way to achieve consensus. One may disagree with his colleague whether a new supplier is "good" or "bad," but it's not difficult to know a new supplier is better (or worse) than the previous supplier in terms of cost, quality, or timeliness (or any other criterion). This situation is usually called multiple attribute decision-making (MADM).

The MADM problem is the decision problem involving a selection of the best alternative(s) from the list of a finite number of alternatives, whereas if one has to design the best alternative, it is called multiple objective decision making (MODM) problem (Javed *et al.*, 2020; Hwang & Yoon, 1981). For instance, a product can be designed in infinite ways, but a customer would only find finite choices in the market. Together MADM and MODM are referred to as multiple criteria decision making (MCDM) or multiple criteria decision analysis. In the current study, our concern is with the MADM problems, or more precisely, the ordinal MADM problems. Unlike the cardinal MADM problem, where the experts express the preference for one alternative over the other using precise numbers, the ordinal MADM problem relies only on the ordinal relations (preference) between them (Contreras, 2010). Such judgements are easier to make than absolute judgements (DCLG, 2009) because most people lack the granulation capacity needed to produce numerically precise information (Danielson & Ekenberg, 2017). Also, in the literature, one can find several ordinal MADM methods however, there is hardly a method like OPA that can estimate the weights of alternatives, criteria, and experts simultaneously while demanding the lowest possible information from the experts. These and other benefits are time and again proven by the studies that have applied the OPA or its variants to solve problems related to supplier selection (Bah & Tulkinov, 2022; Quartey-Papafio *et al.*, 2021), site selection (Popović *et al.*, 2023), process control (Su *et al.*, 2022), service quality (Pitka *et al.*, 2023), wind energy optimization (Ala *et al.*, 2023), evaluations of technologies (Shajedul, 2021) and barriers to technologies (Candra, 2022), optimization of drilling processes (Chakraborty *et al.*, 2023), among others.

The current study gives readers an overview of the OPA in simple language. Maximum effort has been made to avoid scholastic jargons. Also, the study presents the first detailed discussion on the rating scales central to the OPA-based decision-making. The application of the OPA is demonstrated through multiple examples, each representing a different scenario. The codes built on the software LINGO are also provided for the users interested in modifying or extending them as per their needs. The rest of the study is organized as follows. The second section introduces the readers to basic concepts essential to understanding MADM literature. The third section presents the background and the core model of the OPA. The characteristics and framework of the OPA-based decision problem are presented. The OPA scales are discussed. The fourth section includes the application of the OPA on four illustrative examples and some discussion. In the last section, the study is concluded.

2. Basic concepts

Understanding of few concepts is essential before one applies a MADM method. The definitions of these concepts are presented below. If not stated otherwise, these definitions are adapted from Javed *et al.* (2020), in particular, and their predecessors (e.g., Hwang & Yoon, 1981; Kmietowicz & Pearman, 1981; Pearman, 2016) in general.

DEFINITION 1: Alternatives

Alternatives are the entities that are to be evaluated. They may also be known as choices, solutions, options, possibilities, objectives, experiments, states, courses of action, and strategies.

DEFINITION 2: Decision measures

Decision measures are the numerical values that can directly be used to evaluate the alternatives against each decision criterion analytically. When presented together in a table or in the form of a matrix, all decision measures are called payoff matrix or decision matrix, which summarizes the essential information of a decision problem. Generally, linguistic terms and ordinal relations (preferences) cannot be used directly for evaluation as they need to be transformed into numeric values that are directly useable in the MADM method.

DEFINITION 3: Criteria (attributes)

A criterion is a measure of effectiveness (Hwang & Yoon, 1981). It is the yardstick on which the decision alternatives are to be evaluated. Criterion is a general term to represent attribute and objective. They are also called states of nature. Criteria are of different types. Usually, criterion is either positive or negative (and sometimes moderate). Positive criteria are profit-type criteria (higher, the better), e.g., quality, salary, productivity, efficiency, return on investment, etc. Negative criteria are cost-type criteria (lower, the better), e.g., price, defects, infection rate, emissions, etc. Moderate criteria are usually situation specific, e.g., blood pressure (neither too low nor too high is good for health), temperature, and age, among others.

DEFINITION 4: Decision subjects (experts)

The decision subject is the entity that produces the decision measures. In the case of primary data, the respondents, raters or experts are the decision subjects, and in the case of secondary data, the disseminator or generator of the data (e.g., machine, algorithm, report, working paper, organization, newspaper, etc.) is the decision subject. The reliability of decision subjects determines the reliability of decision measures and vice versa. An important factor associated with decision subjects is heterogeneity, which implies the variability between them because of their unique experience, graduation capacity, and domain knowledge (Mahmoudi & Javed, 2023). Superficial differences (e.g., academic titles) should be cautiously included to weigh experts.

DEFINITION 5: Weight

A weight is a measure of relative importance. Since both weights and probabilities vary between 0 and 1, and sums to one, weight can also be conceptualized as probability (Pearman, 2016). In the modern MADM theory such as defined by the OPA, weights are to be estimated of criteria, alternatives, and experts.

DEFINITION 6: Ordinal, scale and relations

Ordinal is a variable/object with an ordered categorical scale (Argesti, 2010), e.g., quality (very good, good, bad, very bad, etc.), cost (very cheap, cheap, expensive, very expensive, etc.), temperature (very high, high, low, very low, etc.) etc. One popular example of ordered categorical scale (or ordinal scale) is Likert scale, a scale that provides a series of close-ended answers extending from one extreme to another (Nemoto & Beglar, 2014). The relations among the values on the ordinal scale are called ordinal paired preferences (ordinal preferences/ ordinal relations/ preference relations). What distinguishes an ordinal relation (A > B) from an equivalence relation (A = B) is asymmetry (Herzberger, 1973; Wiese, 2021), i.e., one is ranked more highly than the other. Meanwhile, what distinguishes ideal experts from other experts is their ability to assign asymmetric relations to all ordinals under study.

3. The ordinal priority approach

3.1 Background

If one has to report a single MADM method that can clearly distinguish the classical MADM theory from the modern one, there is no better name than the Ordinal Priority Approach (OPA). TOPSIS and AHP are the most influential methods of the classical MADM. TOPSIS is a very rational method that evaluates alternatives based on the distance from the best and worst ideal alternatives. Its founders, in their influential work, argued that the "interaction with DM [decision maker]" is "not much" in the [classical] MADM (Hwang & Yoon, 1981). On the other hand, the AHP, a heuristic method (Barzilai, 2001; Munier & Hontoria, 2021) widely portrayed as a scientific (Saaty, 1990), stands on the pairwise comparisons of the alternatives against each attribute and an eigenvector. The quantification of linguistic choices selected by the human experts needed for pairwise comparisons is no easy task (Triantaphyllou & Mann, 1995), and even under the slightest inconsistency in the pairwise comparison matrix, the principal eigenvector components do not give the true relative dominance of the alternatives (Farkas, 2007). Thus, there is no wonder that real-life decision-makers rarely use pairwise comparisons even for simple decision problems ("How many times is design A preferable to design B?"; "twice or thrice?"; "3 times or 3.02 times?"). This, and other aspects of the AHP, have been extensively criticised by practice-oriented researchers (see, e.g., Munier & Hontoria, 2021; Bafahm & Sun, 2019). When compared with other methods, including TOPSIS and AHP, the OPA enjoys several benefits (Ataei *et al.*, 2020), e.g., it interacts with the experts and decision-makers (Mahmoudi & Javed, 2022), its rating scale is flexible, it does not involve normalization of input data (which is one of the first steps in most of the MADM methods), and its algorithm, and thus solution, is not heuristic but optimization. The OPA only requires ranks from the experts as input, and later through a linear programming-based model, it produces the relative weights of not only attributes and alternatives but also of experts. This simultaneous estimation of weights on three dimensions of a MADM problem is a distinguishing feature of the OPA, and is the point where modern MADM theory breaks from the classical one. However, this break is not abrupt but continuous as it holds within it some successes of the classical "ordinal" MADM theory. For instance, the Rank Reciprocal weighting (Stillwell *et al.*, 1981) and the Rank Order Centroid weighting methods (Barron & Barrett, 1996) are the special cases of the OPA for estimating the weights of attributes and alternatives, respectively (Mahmoudi & Javed, 2023). Thus, in the OPA, two competing models, traditionally used for estimating the weights of attributes, complement each other.

3.2 The characteristics and framework

The problems of MADM are diverse, but all problems contain some common characteristics (Hwang and Yoon, 1981; Pearman, 2016) and thus exists a general framework that defines the general characteristics of a decision problem (Kmietowicz & Pearman, 1981). This is the reason one finds the application of MADM methods in very different fields. The common characteristics (and "examples") of an MADM problem are

Multiple alternatives:	"Who are the applicants?"
Multiple attributes (discrete criteria):	"On what attributes are they to be evaluated?"
Multiple experts:	"Who are the evaluators?"
Incommensurable units:	Some attributes (e.g., motivation) are qualitative, some are quantitative (e.g., grades), while some are linguistic (e.g., "excellent", "bad", "not very good", etc.).
Payoff matrix:	"How well each alternative performs against each attribute for each expert?"

In the OPA, the problem of incommensurable units is non-existent because of using ranks as input. Thus, the payoff matrix does not contain numerical scores in incommensurable units. The four characteristics of the OPA-based decision-making are shown in Figure 1, adapted from Ataei *et al.* (2020). The steps involved in the OPA's framework of a decision problem is

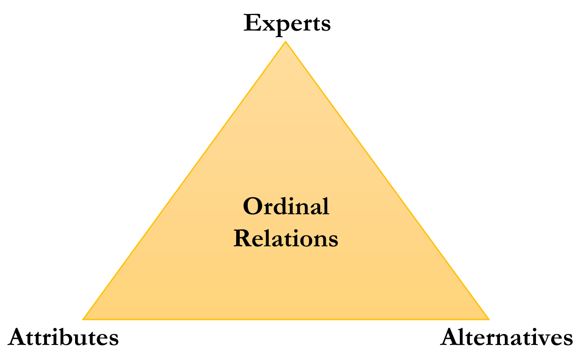

Fig 1. The four characteristics of the OPA-based decision-making (adapted from Ataei *et al.*, 2020)

(i) Identify the goal and the OPA model suitable to achieve it.

(ii) Identify the experts and give them room to stay flexible so they can choose the scale they want and the questions they want to answer.

(iii) Identify the alternatives to be appraised.

(iv) Identify attributes/criteria and their types.

(v) Assess the expected performance of each alternative against each attribute for each expert and the transformation of ordinal relations to numerical values.

(vi) Apply the OPA model and estimate the weights of the experts, attributes, and alternatives.

(vii) Examine the results.

(viii) Sensitivity analysis (confidence level measurement, uncertainty analysis, consistency control charts).

The OPA is not just one model but a complete methodology that includes a set of models that are suitable in different situations, e.g., when input contains fuzziness (subjective uncertainty), the Fuzzy OPA (Mahmoudi *et al.*, 2022) is applicable, and when the input contains greyness (objective uncertainty), the Grey OPA (Mahmoudi *et al.*, 2021) is applicable. Other variants are also available. In the current study, sensitivity analysis will not be performed, and interested readers can refer to Mahmoudi and Javed (2022; 2023).

3.3 The model

Iranian scholar Asgharpour (2003) proposed a model for estimating attribute weights that Amin Mahmoudi later extended in his PhD dissertation to estimate the weights of alternatives and experts as well. The method, as we know today, was first published by Ataei *et al.* (2020). Different variants of the OPA have been proposed since then, e.g., the Fuzzy OPA (Mahmoudi *et al.*, 2022), the Grey OPA (Mahmoudi *et al.*, 2021), the Rough OPA (Du *et al.*, 2023), and the Interval OPA (Mahmoudi *et al.*, 2023), among others. A statistical approach to solve MADM problems with confidence using the OPA has also been developed (Mahmoudi & Javed, 2022).

The basic information needed to read the OPA model are shown below.

Indexes:

i Index of the experts $(1, \dots, p)$

j Index of preference of the attributes $(1, \dots, n)$

k Index of the alternatives $(1, \dots, m)$

Sets:

I Set of experts $\forall i \in$ I

J Set of attributes $\forall j \in$ J

K Set of alternatives $\forall k \in$ K

Parameters:

r_i The rank of expert i

r_j The rank of attribute j

r_k The rank of alternative k

Variables:

Z Objective function

${w_{ijk}}^{r_k}$ Weight (importance) of k^{th} alternative based on j^{th} attribute by i^{th} expert at r_k^{th} rank

Considering the aforementioned information, the computing steps of the OPA are as follows (Mahmoudi & Javed, 2022):

STEP 1. Experts should be identified and ranked based on one or more distinguishing characteristics. Data analyst or decision-maker[1] should rank the experts, if necessary.

STEP 2. In light of the decision-making goal, important attributes should be identified.

STEP 3. Allow each expert to rank the attributes based on their relative importance.

STEP 4. Let the experts rank alternatives based on their relative importance against each attribute.

STEP 5. Using the information from steps 1 to 4, Model (1) should be formed and solved on a computer using some software (such as LINGO, Matlab, Python, etc.).

$$
\begin{aligned}
&Max\ Z \\
&s.t{:} \\
&Z \leq r_i \left(r_j \left(r_k \left(w_{ijk}{}^{r_k} - w_{ijk}{}^{r_k+1} \right) \right) \right) \qquad \forall\, i,j \text{ and } r_k \\
&Z \leq \ r_i r_j r_m w_{ijk}{}^{r_m} \qquad \forall\, i,j \text{ and } r_k = r_m \\
&\sum_{i=1}^{p} \sum_{j=1}^{n} \sum_{k=1}^{m} w_{ijk} = 1 \\
&w_{ijk} \geq 0 \qquad \forall\, i,j \text{ and } k
\end{aligned} \tag{1}
$$

where Z is unrestricted in sign.

After solving Model (1), the alternatives' weight can be calculated using Eq. (2).

$$
w_k = \sum_{i=1}^{p} \sum_{j=1}^{n} W_{ijk} \qquad \forall\, k \tag{2}
$$

To calculate the weights of the attributes, Eq. (3) can be utilized.

$$
w_j = \sum_{i=1}^{p} \sum_{k=1}^{m} W_{ijk} \qquad \forall\, j \tag{3}
$$

In case of need, the experts' weights can be determined by employing Eq. (4).

$$
w_i = \sum_{j=1}^{n} \sum_{k=1}^{m} W_{ijk} \qquad \forall\, i \tag{4}
$$

It should be noted that the OPA allows experts to stay flexible, e.g., it allows them not to answer a question when they lack complete knowledge. Thus, incompleteness in spreadsheet is allowed.

3.4 The scale

The scales of relative preference are well-known in MADM literature (DCLG, 2009) and social and behavioural sciences (e.g., the Likert scale). The OPA involves two scales. One scale (local scale of relative preference) is used to represent the expected performance of each alternative against each attribute for each expert (see Figure 2). It is an ordinal scale that is used to prioritize

[1] In the current study, expert is the one who provides input data while the decision-maker is the end user of the information.

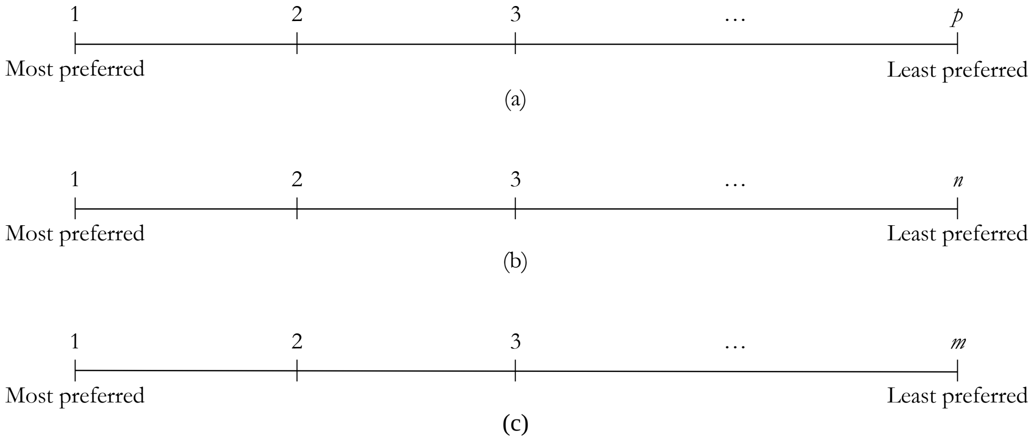

Fig 2. The OPA input: Local scale of relative preference of (a) experts, (b) attributes, and (c) alternatives

the attributes, alternatives, and experts. Thus, it is used to represent the input. The second scale (global scale of relative preference) is used to examine the results (see Figure 3). By multiplying the scores on this later scale with 100, one can express the relative strength of preferences as a percentage. In short, the OPA represents the procedure of transforming the local scales of relative preference to global ones.

The number of points on the OPA scale cannot exceed the number of objects that needs to be relatively prioritized. For instance, if there are nine attributes and one has no other information except for their relative preference, the scale must not exceed nine, i.e., ideally, 1 for the most important attribute and 9 for the least important attribute. The OPA scale shares some resemblance with the Likert scale and Borgatta and Bohrnstedt (1980)'s 'imperfect interval scale.' However, unlike the Likert scale and Saaty's 9-point scale for the AHP, the OPA scale's upper limit varies from one expert to another and, thus, from one problem to another (dynamic scaling). For instance, a more knowledgeable expert may assign a unique rank to each object (attribute or alternative) (by using a longer scale), while a less knowledgeable expert may assign the same ranks (ties) to some objects.

Psychological experiments have shown that individuals cannot simultaneously compare more than seven objects (plus or minus two) (Miller, 1956). That's why one frequently see 7 or 9-point Likert scales in questionnaires (Brunt *et al.*, 2022; Sahashi *et al.*, 2021), and this was the main reason Saaty also chose 9 as the upper limit of the AHP scale (Triantaphyllou & Mann, 1995). However, in the OPA, there is no upper limit on the choice of the scale, but ideally, each object is assigned a unique rank by each expert. As human granulation capacity is limited and thus in large-scale problems (e.g., a problem containing 50 attributes), one may argue that it is unrealistic to expect the experts to assign unique ranks to each attribute precisely. In such cases, user-defined constraints can be imposed on the OPA's local scales (Figure 2) to produce the likes of the OPA scale, as shown in Figure 4. Figure 4 shows the classical (Hwang & Yoon, 1981) and constrained OPA scales for cost- and profit-type attributes. The OPA scale in the table can be extended to 11 or more values as well based on the granulation capacity of the expert(s).

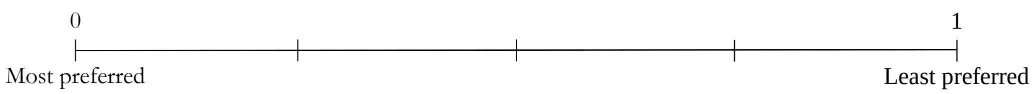

Fig 3. The OPA output: Global scale of relative preference (weight)

Interval scale in the classical MADM				**The nine-point OPA scale**			
Profit			Cost	Profit			Cost
Very low	1	1	*Very high*	*Very profitable*	1	1	*Very uncostly*
	2	2			2	2	
Low	3	3	*High*	*Profitable*	3	3	*Uncostly*
	4	4			4	4	
Average	5	5	*Average*	*Average*	5	5	*Average*
	6	6			6	6	
High	7	7	*Low*	*Unprofitable*	7	7	*Costly*
	8	8			8	8	
Very high	9	9	*Very low*	*Very unprofitable*	9	9	*Very costly*

Fig 4. Assignment of nine values for a rating scales in the classical and OPA-based MADM

4. Application

4.1 Software

There are several ways to execute the OPA model. One way is using the OPA Solver, an indigenous software developed and maintained by Amin Mahmoudi, and another way is coding on computer programs such as on LINGO, Python, Mathematica, and MATLAB, among others. In the current study, we will use LINGO as running optimization models on it is much easier than other software, and users can very easily edit and modify the codes as per their requirements. To execute the OPA model on LINGO one only needs the knowledge of the operators presented in Table 1. Figure 5 shows how a model looks like in LINGO. If one presses the direct hit (bullseye) icon, one gets the result (as shown in Figure 6).

4.2 Illustrative examples

In the following sub-sections, four hypothetical cases represent four ways the OPA can be applied. In real life, many more cases are possible.

EXAMPLE 1 – Decision making with one expert

Let us assume we have three suppliers (A1, A2 and A3) who are to be evaluated based on three attributes – quality (C1), cost (C2), and resilience (C3). Time was limited; hence procurement manager himself decided to evaluate them based on his experience with the suppliers. The payoff matrix produced by the manager is shown in Table 2.

The manager asked an intern working under him to solve the problem who decided to solve the problem using the OPA. The code she typed on LINGO is shown in *Appendix A*. In the code, the number outside the opening round bracket shows the rank of the attribute(s), while the first number inside the opening round bracket represents the rank of alternative(s). The results of the model are shown in Table 3. Thus, in her report to the manager, the intern mentioned that the first supplier (A1) was best as it scored the highest relative weight (0.411). The report also noted that the first attribute (C1) is the most important as it scored the highest weight (0.480). The ranking of the attributes and alternatives is shown in Table 4.

EXAMPLE 2 – Decision making with two experts

In her report, the intern suggested that if the opinions of more experts were sought, the results could be more reliable. The procurement manager (E1) liked the suggestion and asked her to collect data from a procurement assistant (E2), who was also well aware of the three suppliers. The intern collected data from the said individual; however, after talking with the assistant, she noticed that he was less knowledgeable than the manager, and thus he ranked the two experts as E1 > E2. The opinions of both experts are shown in Table 5. One can see that E2 is confused at one point (C3:A1/A2) and lacks information at another point (C2:A2).

Table 1. The LINGO operators

Sign	Operator	Sign	Operator
*	Multiplication	=	Equal to
+	Addition	>	Greater than
-	Subtraction	>=	Greater than and equal to
(	Opening round bracket	)	Closing round bracket
;	Semicolon (sentence break)	!	Exclamation mark

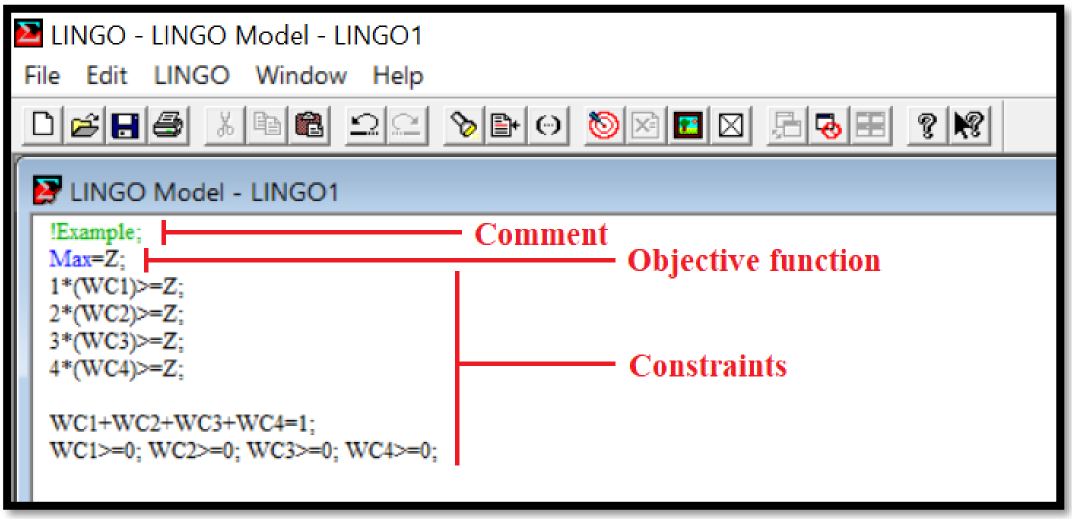

Fig 5. Writing the OPA model in LINGO

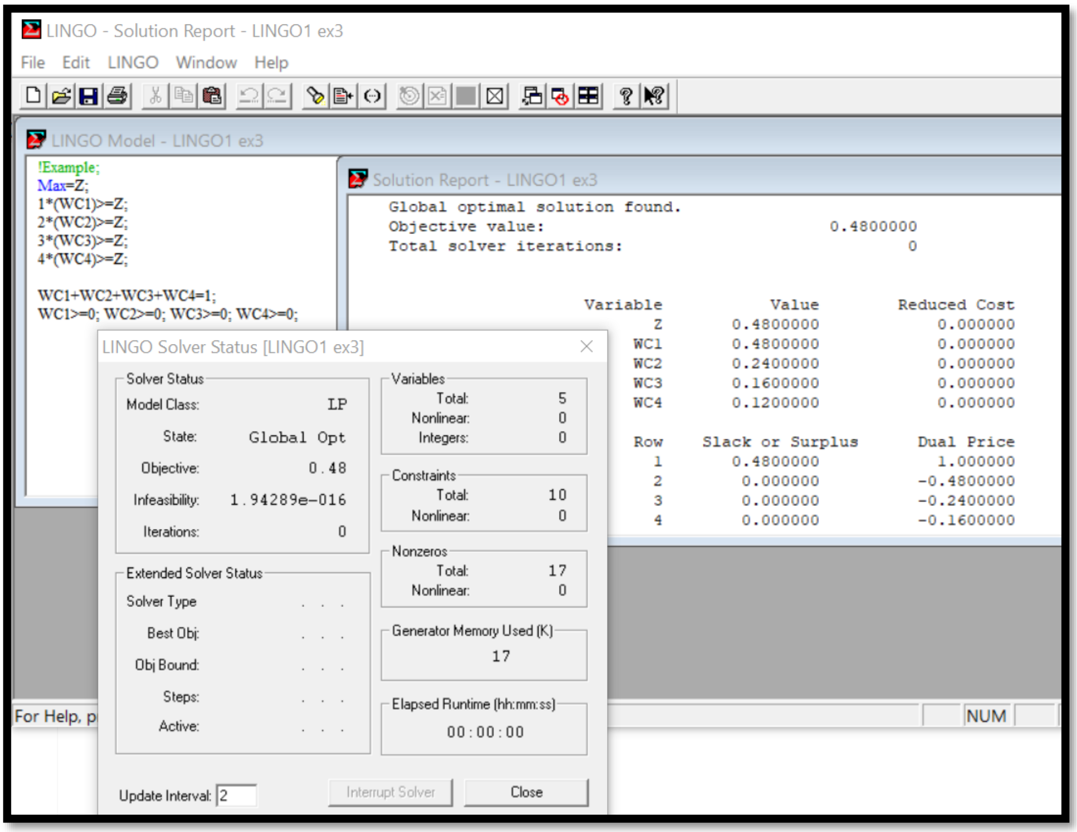

Fig 6. A sample of the OPA model in LINGO and its results

Table 2. The ordinal payoff matrix – Example 1

	1	2	3	4
	C1	**C2**	**C3**	**C4**
A1	1	3	3	1
A2	2	2	1	3
A3	3	1	2	2

Table 3. The local and global weights of the alternatives and attributes – Example 1

WC1A1	0.2933	WC2A1	0.0267	WC3A1	0.0178	WC4A1	0.0733	**W(A1)**	**0.411**
WC1A2	0.1333	WC2A2	0.0667	WC3A2	0.0978	WC4A2	0.0133	**W(A2)**	**0.311**
WC1A3	0.0533	WC2A3	0.1467	WC3A3	0.0444	WC4A3	0.0333	**W(A3)**	**0.278**
W(C1)	**0.480**	**W(C2)**	**0.240**	**W(C3)**	**0.160**	**W(C4)**	**0.120**		
NOTE: Local weights are not bold while global weights are bold. Rows represent attributes and columns represent alternatives									

Table 4. Ranks of the attributes and alternatives – Example 1

Index	Weight	Rank
C1	0.480	1
C2	0.240	2
C3	0.160	3
C4	0.120	4
A1	0.411	1
A2	0.311	2
A3	0.278	3

Table 5. The ordinal payoff matrix – Example 2

	1				2			
	E1				**E2**			
	1	2	3	4	1	3	2	4
	C1	**C2**	**C3**	**C4**	**C1**	**C2**	**C3**	**C4**
A1	1	3	3	1	1	1	1	1
A2	2	2	1	3	2		1	3
A3	3	1	2	2	3	2	2	2

The intern retyped the OPA model in LINGO, and the resultant code is shown in *Appendix B*. In the code, the first number outside the opening round bracket shows the rank of expert, and the second shows the rank of the attribute, while the first number inside the opening round bracket represents the rank of alternative. If there was no empty box in Table 5, the intern was supposed to mention WE2C2A2 as well among the last two constraints (= 1, and >=0). The final results of the model are shown in Table 6. Thus, in her new report to the manager, the intern mentioned that the first supplier (A1) is still the best as it scored the highest relative weight (0.465). The report also noted that the first attribute (C1) is still the most important, as it scored the highest weight (0.482). Since it was a group decision-making problem, the results are more likely to be perceived as reliable than the one reported in Example 1.

EXAMPLE 3 – Estimation of weights of the attributes (single expert)

Let us assume the intern wants to use another method for ranking the suppliers for the sake of comparative analysis. Since the method cannot estimate the attribute weights, she needs to find a way to get these weights. She believes the ranking of attributes produced by the first expert (E1)

Table 6. Ranks of the attributes and alternatives – Example 2

Index	Weight	Rank
E1	0.670	1
E2	0.330	2
C1	0.482	1
C2	0.196	3
C3	0.201	2
C4	0.121	4
A1	0.465	1
A2	0.298	2
A3	0.237	3

to be the most reliable, and thus now needs to estimate the attribute weights. The data is shown in Table 7.

The intern typed the OPA model in LINGO, and the resultant code is shown in *Appendix C*. In the code, the first number outside the opening round bracket shows the rank of the attribute. The final results of the model are shown in Table 8.

EXAMPLE 4 – Estimation of weights of the attributes (two experts)

To make the analysis more reliable, the intern also included the opinions of the second expert (E2) from Example 3. The data is shown in Table 9.

The intern retyped the OPA model in LINGO, and the resultant code is shown in *Appendix D*. In the code, the first number outside the opening round bracket shows the rank of expert, and the second shows the rank of the attribute. The final results are shown in Table 10. One could also get the weights of the experts from the same code (E1 = 0.667; E2 = 0.333); however, it was not needed in this example and thus is not shown in the table.

5. Conclusion

The article shows the background, methodology, and execution of the OPA. The intended audience is the students of multiple-attribute decision-making. The importance of the rating scale in the OPA is highlighted. Through four easy-to-understand illustrative examples, the execution of the OPA using LINGO software is presented. The OPA has received considerable attention from researchers in a short period. Considering the rising interest in the OPA since its introduction in 2020, it was needed that the method is straightforwardly explained to the general public and interested users in the industry. The readers would find the article easy to follow and useful for solving real-life problems. With the aid of the given examples, the readers can model their own problems using the OPA on LINGO.

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Table 7. The ranking of attributes – Example 3

C1	C2	C3	C4
1	2	3	4

Table 8. Weights (against the corresponding ranks) of the attributes – Example 3

Index	Weight	Rank
C1	0.480	1
C2	0.240	2
C3	0.160	3
C4	0.120	4

Table 9. The ranking of the attributes – Example 4

E1				E2			
1				2			
C1	C2	C3	C4	C1	C2	C3	C4
1	2	3	4	1	3	2	4

Table 10. Weights (against the corresponding ranks) of the attributes – Example 4

Index	Weight	Rank
C1	0.480	1
C2	0.213	2
C3	0.187	3
C4	0.120	4

Appendix A: The LINGO code associated with Example 1

```
Max=Z;
1*(1*(WC1A1-WC1A2))>=Z;
1*(2*(WC1A2-WC1A3))>=Z;
1*(3*(WC1A3))>=Z;

2*(1*(WC2A3-WC2A2))>=Z;
2*(2*(WC2A2-WC2A1))>=Z;
2*(3*(WC2A1))>=Z;

3*(1*(WC3A2-WC3A3))>=Z;
3*(2*(WC3A3-WC3A1))>=Z;
3*(3*(WC3A1))>=Z;

4*(1*(WC4A1-WC4A3))>=Z;
4*(2*(WC4A3-WC4A2))>=Z;
4*(3*(WC4A2))>=Z;

WC1A1+WC1A2+WC1A3+WC2A1+WC2A2+WC2A3+WC3A1+WC3A2+WC3A3+WC4A1+WC4A2+WC4A3=1;

WC1A1>=0;WC1A2>=0;WC1A3>=0;WC2A1>=0;WC2A2>=0;WC2A3>=0;WC3A1>=0;WC3A2>=0;WC3A3>=0;WC4A1>=0;WC
4A2>=0;WC4A3>=0;
```

Appendix B: The LINGO code associated with Example 2

```
Max=Z;
1*1*(1*(WE1C1A1-WE1C1A2))>=Z;
1*1*(2*(WE1C1A2-WE1C1A3))>=Z;
1*1*(3*(WE1C1A3))>=Z;

1*2*(1*(WE1C2A3-WE1C2A2))>=Z;
1*2*(2*(WE1C2A2-WE1C2A1))>=Z;
1*2*(3*(WE1C2A1))>=Z;

1*3*(1*(WE1C3A2-WE1C3A3))>=Z;
1*3*(2*(WE1C3A3-WE1C3A1))>=Z;
1*3*(3*(WE1C3A1))>=Z;

1*4*(1*(WE1C4A1-WE1C4A3))>=Z;
1*4*(2*(WE1C4A3-WE1C4A2))>=Z;
1*4*(3*(WE1C4A2))>=Z;

2*1*(1*(WE2C1A1-WE2C1A2))>=Z;
2*1*(2*(WE2C1A2-WE2C1A3))>=Z;
2*1*(3*(WE2C1A3))>=Z;

2*3*(1*(WE2C2A1-WE2C2A3))>=Z;
2*3*(2*(WE2C2A3))>=Z;

2*2*(1*(WE2C3A1-WE2C3A3))>=Z;
2*2*(1*(WE2C3A2-WE2C3A3))>=Z;
2*2*(2*(WE2C3A3))>=Z;

2*4*(1*(WE2C4A1-WE2C4A3))>=Z;
2*4*(2*(WE2C4A3-WE2C4A2))>=Z;
2*4*(3*(WE2C4A2))>=Z;
```

```
WE1C1A1+WE1C1A2+WE1C1A3+WE1C2A1+WE1C2A2+WE1C2A3+WE1C3A1+WE1C3A2+WE1C3A3+WE1C4A1+WE1C4A2+WE1C
4A3+WE2C1A1+WE2C1A2+WE2C1A3+WE2C2A1+WE2C2A3+WE2C3A1+WE2C3A2+WE2C3A3+WE2C4A1+WE2C4A2+WE2C4A3=
1;

WE1C1A1>=0; WE1C1A2>=0; WE1C1A3>=0; WE1C2A1>=0; WE1C2A2>=0; WE1C2A3>=0; WE1C3A1>=0;
WE1C3A2>=0; WE1C3A3>=0; WE1C4A1>=0; WE1C4A2>=0; WE1C4A3>=0;

WE2C1A1>=0; WE2C1A2>=0; WE2C1A3>=0; WE2C2A1>=0; WE2C2A3>=0; WE2C3A1>=0; WE2C3A2>=0;
WE2C3A3>=0; WE2C4A1>=0; WE2C4A2>=0; WE2C4A3>=0;
```

Appendix C: The LINGO code associated with Example 3

```
Max=Z;
1*(WC1)>=Z;
2*(WC2)>=Z;
3*(WC3)>=Z;
4*(WC4)>=Z;

WC1+WC2+WC3+WC4=1;
WC1>=0; WC2>=0; WC3>=0; WC4>=0;
```

Appendix D: The LINGO code associated with Example 4

```
Max=Z;
1*1*(WE1C1)>=Z;
1*2*(WE1C2)>=Z;
1*3*(WE1C3)>=Z;
1*4*(WE1C4)>=Z;

2*1*(WE2C1)>=Z;
2*3*(WE2C2)>=Z;
2*2*(WE2C3)>=Z;
2*4*(WE2C4)>=Z;

WE1C1+WE1C2+WE1C3+WE1C4+WE2C1+WE2C2+WE2C3+WE2C4=1;
WE1C1>=0; WE1C2>=0; WE1C3>=0; WE1C4>=0;
WE2C1>=0; WE2C2>=0; WE2C3>=0; WE2C4>=0;
```

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