

Analytical Ordinal Priority Approach

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Abstract: The study proposes an analytical (closed-form) solution to the Ordinal Priority Approach (OPA) in multiple attribute decision-making. The proposed Analytical Ordinal Priority Approach (AOPA) can calculate the weights of alternatives, criteria and experts, without linear programming. The application of the AOPA is demonstrated through an example run on Microsoft Excel. The results are consistent with those of the classical OPA. The findings are important for those who seek convenience and may wish to execute the OPA on commonly used spreadsheets without the need for programming languages.

Keywords: Ordinal Priority Approach; analytical; closed-form; multiple criteria decision analysis

1. Introduction

Multiple criteria decision analysis (MCDA) is an important part of operations research and decision theory with applications in numerous disciplines. It provides a structured framework for analysing decision-making problems characterized by complex multiple objectives (Ananda & Herath, 2009). Even though the MCDA problems are diverse they share some common characteristics, e.g., multiple criteria (objectives or attributes), conflict among criteria, incommensurable units and design or selection (Hwang & Yoon, 1991). Most MCDA methods are designed to rank alternatives against conflicting attributes, such as the Grey Relational Analysis (GRA), Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), among others. Most MCDA methods need third-party techniques for estimating the weights of the attributes such as the Rank Order Centroid method (Barron & Barrett, 1996), the Rank Reciprocal method (Stillwell *et al.*, 1981), the Entropy method (Mukhametzhanov, 2021), among others.

When a decision-making problem involves inputs from multiple experts such problems are called multiple criteria group decision-making (MCGDM), or group decision-making (GDM). If one look at numerous literatures on MCGDM, one finds that, despite the important role expert opinions play in these problems, experts are rarely weighed. A common strategy for such problems is the aggregation of the expert judgements using arithmetic or geometric means (Saaty & Vargas, 2007). Thus, integrating the expert weighting mechanism in the theory of MCGDM was one of the least explored areas within the continuously growing scholarship on the MCGDM. In 2020, the Ordinal Priority Approach (OPA) was released that solved the major problem of simultaneous estimation of the weights of experts, attributes and alternatives. Thus, an increasing number of

studies are recognizing the OPA as a breakthrough methodology in the field (Aouadni *et al.*, 2024; Čačić *et al.*, 2024; Wang *et al.*, 2022) whereas in just a short span of time, this methodology has seen multiple extensions (Debroy *et al.*, 2025; Du *et al.*, 2024) and several applications in different fields (see, e.g., Pitka *et al.*, 2023; Bah & Tulkinov, 2022; Kiptum *et al.*, 2022).

The OPA is a linear programming-based technique and it requires a computer program (e.g., LINGO, Python, MATLAB, Wolfram Mathematica, etc.) to smoothly execute it. Even though the OPA is garnering increasing recognition with each passing day, it has been observed that there is an immediate need for an analytical (closed-form) solution to the OPA so it can be applied readily and conveniently on popular spreadsheets (e.g., Microsoft Excel, Apple Numbers, Google Sheets, WPS Office Spreadsheet, etc.) available in the computers nowadays around the world. Based on the profound experience of developing and applying the OPA and viewing its applications by other scholars in the market, the authors of the current study have amassed rich insights on the functioning of the OPA. Guided by these insights and observations, along with some empirical evidences, the current study proposes analytical solutions to the OPA to solve the MCGDM problems.

The rest of the study is organized as follows: the next section presents the background that led to the development of the proposed analytical methods for estimating the weights of experts, attributes and alternatives. This section is followed by a section where the proposed system of equations, called Analytical Ordinal Priority Approach (AOPA), is presented. In the subsequent section, the proposed technique is applied on a hypothetical example. Lastly, the study is concluded with some important takeaways.

2. Background

In 2020, the Ordinal Priority Approach was published by a team led by Amin Mahmoudi (Ataei *et al.*, 2020). The OPA method determines the individual weights w_{ijk} by maximizing the objective function Z , which incorporates the ranks of alternatives, attributes and experts. These weights are then summed up to obtain the aggregated weights for alternatives, attributes, and experts. The basic information needed to read the OPA model are shown below.

INDEXES:

- i Index of the experts $(1, \dots, p)$
- j Index of preference of the attributes $(1, \dots, n)$
- k Index of the alternatives $(1, \dots, m)$

SETS:

- I Set of experts $\forall i \in I$
- J Set of attributes $\forall j \in J$
- K Set of alternatives $\forall k \in K$

PARAMETERS:

- r_i The rank of expert i
- r_j The rank of attribute j
- r_k The rank of alternative k

VARIABLES:

- Z Objective function
- $w_{ijk}^{r_k}$ Weight (importance) of k^{th} alternative based on j^{th} attribute by i^{th} expert at r_k^{th} rank

The following linear programming model represents the classical OPA and is supposed to be solved using a programming language (Mahmoudi & Javed, 2023a),

$$\begin{aligned}
 & \text{Max } Z \\
 & \text{s. t:} \\
 & Z \leq r_i \left(r_j \left(r_k \left(W_{ijk}^{r_k} - W_{ijk}^{r_{k+1}} \right) \right) \right) \quad \forall i, j \text{ and } r_k \\
 & Z \leq r_i r_j r_m W_{ijk}^{r_m} \quad \forall i, j \text{ and } r_k = r_m \\
 & \sum_{i=1}^p \sum_{j=1}^n \sum_{k=1}^m w_{ijk} = 1 \\
 & W_{ijk} \geq 0 \quad \forall i, j \text{ and } k
 \end{aligned} \tag{1}$$

where Z is unrestricted in sign.

After solving Model (1), the experts' weights can be determined by employing Eq. (2).

$$W_i = \sum_{j=1}^n \sum_{k=1}^m W_{ijk} \quad \forall i \tag{2}$$

To calculate the weights of the attributes, Eq. (3) can be utilized.

$$W_j = \sum_{i=1}^p \sum_{k=1}^m W_{ijk} \quad \forall j \tag{3}$$

And, the alternatives' weight can be calculated using Eq. (4).

$$W_k = \sum_{i=1}^p \sum_{j=1}^n W_{ijk} \quad \forall k \tag{4}$$

Mahmoudi and Javed (2023a) extended the OPA through interval mathematics. In their work some important findings were reported related to the theory of the OPA. They illustrated that as the number of objects (e.g., attributes or alternatives) to be ranked increases, the difference in importance between them gets smaller as one moves from top ranked objects to lower ranked objects. It means, the difference between the ranks 1 and 2 is larger than the difference between the ranks 2 and 3, and so on. There are several rank-based methods that exhibit these properties. Further, they stated that,

“In fact, two of the rank-based methods—rank reciprocal (Stillwell *et al.* 1981) and rank order centroid (Barron and Barrett 1996)—are special cases of the Ordinal Priority Approach on the criterion weighting and alternative weighting dimensions, respectively.”

Therefore, “in the OPA, two competing models, traditionally used for estimating the weights of attributes, complement each other” (Javed & Du, 2023). In Proposition 2 and Definition 2 of Mahmoudi and Javed (2023a), they argued that the Rank Reciprocal method is a special case of the OPA for estimating the weights of attributes (criteria), when all experts are equally important or when there is only one expert. Based on this construct later they derived important results.

In their Proposition 1 and Definition 1, they argued that the Rank Order Centroid method is a special case of the OPA for estimating the weights of alternatives, when all experts are equally important or when there is only one expert. Meanwhile, it should be noted that in the classical OPA (Ataei *et al.*, 2020), weight (importance) of k^{th} alternative is not absolute, but is defined relative to j^{th} criterion and i^{th} expert at r^{th} rank. Actually, the first constraint of the OPA model indirectly manifests this construct. If one look at the first inequality of the OPA model,

$$Z \leq r_i \left(r_j \left(r_k \left(W_{ijk}^{r_k} - W_{ijk}^{r_{k+1}} \right) \right) \right) \quad \forall i, j \text{ and } r_k \quad (5)$$

one can observe that the importance of an alternative at a given rank, based on a specific expert and attribute, is tied to the expert's rank (r_i) and the attribute's rank (r_j). The higher the ranks of the expert and attribute, the more significant the difference in weights between consecutive alternative ranks for Z . Similar conclusions can be drawn from the careful reading of Mahmoudi and Javed (2023b). Also, as the core decision variable, $W_{ijk}^{r_k}$, is the weight (importance) of k^{th} alternative based on j^{th} attribute by i^{th} expert at r_k^{th} rank therefore, one can argue that the weight of alternative is the function of the rank of alternatives as well as the rank (and thus, weight) of attribute and rank (and, thus, weight) of expert, i.e., at the r_k^{th} rank,

$$W_k = f(r_k, r_j, r_i), \quad (6)$$

or, more precisely,

$$W_{ijk} = f(r_{ijk}, r_{ij}, r_i), \quad (7)$$

Meanwhile, in another work (Mahmoudi & Javed, 2022), they clearly argued that the qualification of experts is the prerequisite to the qualification of attributes (criteria), which in turns is a prerequisite to the qualification of alternatives. Thus, in the OPA the weights are hierarchically determined, i.e., each object's weight (importance) is influenced by its position relative to other ranked objects. Mahmoudi and Javed (2023a) defined the weight estimated through the OPA as a probability of a given object's priority over the other. Thus, in the OPA, it's common to write "weights (importance)" (Ataei *et al.*, 2020) because the OPA weights are not necessarily the "weights." Depending on a situation, they can denote probabilities (Mahmoudi & Javed, 2023a; Javed & Du, 2023) or something else as well. Based on the authors' understanding of the behaviour of the weights of the OPA (as the number of objects increase), and the properties of the OPA, three axioms and few propositions are advanced in the current study:

AXIOM 1: Weight (Expert) = f (Rank (Expert)).

AXIOM 2: Weight (Attribute) = f (Rank (Attribute), Rank (Expert)).

AXIOM 3: Weight (Alternative) = f (Rank (Alternative), Rank (Attribute), Rank (Expert)).

These three axioms are proven from the discussion that preceded them.

PROPOSITION 1: In the OPA, a "weight" is a unit interval value (or scaled value) that represents the relative behaviour of ranked objects. A "weight" in one case may be conceptualized as a "probability" in another case and an index (or score) of relative importance (or performance) in another case.

It is proven from literature (Mahmoudi & Javed, 2023a; Javed & Du, 2023).

PROPOSITION 2: In the OPA, the weights (or importance) are hierarchically determined i.e., the position of objects (alternative, attribute, and expert) relative to each other matters.

It is proven from Axioms 1 to 3.

3. Analytical Ordinal Priority Approach

The Analytical Ordinal Priority Approach (AOPA) is the analytical equivalent of the classical linear programming-based Ordinal Priority Approach (OPA). Given the data is complete and there is no tie, in this approach the weights of the experts, attributes and alternatives would be calculated using the following definitions.

DEFINITION 1: Expert weights

In a three-dimensional multiple attribute group decision making problem, if r_i is the rank of i^{th} expert and total number of experts are p , then the weight of i^{th} expert will be given as

$$W_i = \frac{\frac{1}{r_i}}{\sum_{i=1}^p \frac{1}{r_i}} \quad (8)$$

These weights are absolutely consistent with the weights calculated using the Rank Reciprocal method, if applied on experts.

DEFINITION 2: Attribute weights

In a multiple attribute group decision making problem, if r_i is the rank of i^{th} expert, and r_{ij} is the rank of j^{th} attribute against i^{th} expert whereas, the total number of experts are p and the total number of attributes are n , then the weight of j^{th} attribute will be given as

$$W_j = \frac{u_j}{\sum_{j=1}^n u_j} \quad (9)$$

where,

$$u_j = \sum_{i=1}^p \left(\frac{1}{r_i r_{ij}} \right) \quad (10)$$

or, simply,

$$W_j = \frac{\sum_{i=1}^p \left(\frac{1}{r_i r_{ij}} \right)}{\sum_{j=1}^n \left(\sum_{i=1}^p \left(\frac{1}{r_i r_{ij}} \right) \right)} \quad (11)$$

The attribute weight estimation method is a direct generalization of the expert weight estimation method. If a problem involves only one expert (or all experts are equally important), the formula of the attribute weight would have a structure similar to that of the expert weight.

DEFINITION 3: Alternative weights

In a multiple attribute group decision making problem, let us assume that r_i is the rank of i^{th} expert, and r_{ij} is the rank of j^{th} attribute against i^{th} expert and r_{ijk} is the rank of k^{th} alternative against j^{th} attribute assigned by i^{th} expert. If the total number of experts are p , the total number of attributes are n , and the total number of alternatives are m , then the weight of k^{th} alternative will be given as,

$$W_k = \frac{v_k}{\sum_{k=1}^m v_k} \quad (12)$$

where,

$$v_k = a_{1jk} + a_{2jk} + \dots + a_{pjk} = \sum_{i=1}^p a_{ijk} \quad (13)$$

where,

$$a_{ijk} = \sum_{j=1}^n \left(\frac{1}{r_i r_{ij}} \times \sum_{r_{ijk}=k}^p \frac{1}{r_{ijk}} \right) \quad (14)$$

It should be noted that a_{ijk} is a very interesting coefficient. On right hand side, the first part $\left(\frac{1}{r_i r_{ij}}\right)$ is inspired by the rank reciprocal operation of the Rank Reciprocal method while the second part $\left(\sum_{r_{ijk}=k}^p \frac{1}{r_{ijk}}\right)$ is inspired by the rank aggregation operation of the Rank Order Centroid method. Thus, a_{ijk} represents a novel contribution of the AOPA to the decision theory. In short, the weight of k^{th} alternative will be given as,

$$W_k = \frac{\sum_{i=1}^p \left(\sum_{j=1}^n \left(\frac{1}{r_i r_{ij}} \times \sum_{r_{ijk}=k}^p \frac{1}{r_{ijk}} \right) \right)}{\sum_{k=1}^m \left(\sum_{i=1}^p \left(\sum_{j=1}^n \left(\frac{1}{r_i r_{ij}} \times \sum_{r_{ijk}=k}^p \frac{1}{r_{ijk}} \right) \right) \right)} \quad (15)$$

The alternative weight estimation method is a complex generalization of the attribute weight estimation method. When we have one expert (or all experts are equally important), and one attribute (or all attributes are equally important), the formula of the alternative weight would have a structure similar to that of the attribute weight.

Now another exercise can be done, for the sake of convenience of our readers who want to apply the AOPA with further ease. If we assume that

$$g = \sum_{r_{ijk}=k}^p \frac{1}{r_{ijk}} \quad (16)$$

Then a g -score table can be constructed for quick reference. The g -scores represent transformation of the ranks r_{ijk} . The g -score table for up to twenty alternatives is shown in *Table 1*. *Table 1* can be used for any group decision-making problem that involves two to twenty alternatives. For larger problems, it can be extended by using Eq. (16). It should be noted that the sum of each column containing the g -scores equals the number of alternatives. Those users who may need to extend this table, can use this point to double check their calculations.

4. Application

In this section the AOPA and the OPA will be applied on a hypothetical case involving three experts ($p = 3$), four attributes ($n = 4$), and five alternatives ($m = 5$).

4.1 Calculating weights of the experts

It is believed that the first expert (E1) is considered more authoritative than the second expert (E2), who is considered more authoritative than the third expert (E3), i.e.,

$$E1 > E2 > E3.$$

Thus, by applying Eq. (8), the results that we got are shown in *Table 2*. For comparative analysis, the OPA weights are also shown in the last column of the table. One can see that the expert weights obtained through the AOPA are exactly same like those obtained through the OPA. The first expert got 54.5% weight, while the second and third experts got 27.3% and 18.2%, respectively.

Table 1. The table of g-scores

m^{-1}	m	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1.0 00	1	1.5 00	1.8 33	2.0 83	2.2 83	2.4 50	2.5 93	2.7 18	2.8 29	2.9 29	3.0 20	3.1 03	3.1 80	3.2 52	3.3 18	3.3 81	3.4 40	3.4 95	3.5 48	3.5 98
0.5 00	2	0.5 00	0.8 33	1.0 83	1.2 83	1.4 50	1.5 93	1.7 18	1.8 29	1.9 29	2.0 20	2.1 03	2.1 80	2.2 52	2.3 18	2.3 81	2.4 40	2.4 95	2.5 48	2.5 98
0.3 33	3		0.3 33	0.5 83	0.7 83	0.9 50	1.0 93	1.2 18	1.3 29	1.4 29	1.5 20	1.6 03	1.6 80	1.7 52	1.8 18	1.8 81	1.9 40	1.9 95	2.0 48	2.0 98
0.2 50	4			0.2 50	0.4 50	0.6 17	0.7 60	0.8 85	0.9 96	1.0 96	1.1 87	1.2 70	1.3 47	1.4 18	1.4 85	1.5 47	1.6 06	1.6 62	1.7 14	1.7 64
0.2 00	5				0.2 00	0.3 67	0.5 10	0.6 35	0.7 46	0.8 46	0.9 37	1.0 20	1.0 97	1.1 68	1.2 35	1.2 97	1.3 56	1.4 12	1.4 64	1.5 14
0.1 67	6					0.1 67	0.3 10	0.4 35	0.5 46	0.6 46	0.7 37	0.8 20	0.8 97	0.9 68	1.0 35	1.0 97	1.1 56	1.2 12	1.2 64	1.3 14
0.1 43	7						0.1 43	0.2 68	0.3 79	0.4 79	0.5 70	0.6 53	0.7 30	0.8 02	0.8 68	0.9 31	0.9 90	1.0 45	1.0 98	1.1 48
0.1 25	8							0.1 25	0.2 36	0.3 36	0.4 27	0.5 10	0.5 87	0.6 59	0.7 25	0.7 88	0.8 47	0.9 02	0.9 55	1.0 05
0.1 11	9								0.1 11	0.2 11	0.3 02	0.3 85	0.4 62	0.5 34	0.6 00	0.6 63	0.7 22	0.7 77	0.8 30	0.8 80
0.1 00	10									0.1 00	0.2 91	0.3 74	0.4 51	0.4 23	0.5 89	0.5 52	0.6 11	0.6 66	0.7 19	0.7 69
0.0 91	11										0.0 91	0.1 74	0.2 51	0.3 23	0.3 89	0.4 52	0.5 11	0.5 66	0.6 19	0.6 69
0.0 83	12											0.0 83	0.1 60	0.2 32	0.2 98	0.3 61	0.4 20	0.4 75	0.5 28	0.5 78
0.0 77	13												0.0 77	0.1 48	0.2 15	0.2 78	0.3 36	0.3 92	0.4 45	0.4 95
0.0 71	14													0.0 71	0.1 38	0.2 01	0.2 59	0.3 15	0.3 68	0.4 18
0.0 67	15														0.0 67	0.1 29	0.1 88	0.2 44	0.2 96	0.3 46
0.0 63	16															0.0 63	0.1 21	0.1 77	0.2 30	0.2 80
0.0 59	17																0.0 59	0.1 14	0.1 67	0.2 17
0.0 56	18																	0.0 56	0.1 08	0.1 58
0.0 53	19																		0.0 53	0.1 03
0.0 50	20																			0.0 50

Table 2. The estimation of expert weights using Analytical OPA

	Rank	$1/r_i$	W_i (AOPA)	W_i (OPA)
E1	1	1.000	0.545	0.545
E2	2	0.500	0.273	0.273
E3	3	0.333	0.182	0.182

4.2 Calculating weights of the attributes

Each expert ranked the four attributes (C1, C2, C3, C4) differently. For instance, for the first expert, the first attribute is more important than the second attribute, which is more important than the third attribute, which in turns is considered least important.

$$C1 > C2 > C3 > C4.$$

While for the second expert,

$$C4 > C3 > C2 > C1$$

and for the third expert,

$$C4 > C1 > C2 > C3.$$

These ranks are shown in Table 3, along with the results obtained through the applications of Eqs. (9) and (10). One can see that the attribute weights obtained through the AOPA are consistent with those obtained through the OPA. It is found that the first attribute is most important with

33.8% weight, while the fourth, third and second attributes got 28.4%, 20.4% and 17.5% weights, respectively. Thus, overall,

$$C1 > C4 > C2 > C3.$$

4.3 Calculating weights of the alternatives

In the end, each expert evaluated each of the five alternatives (A1, A2, A3, A4, A5) against each attribute, and the decision matrix is shown in Table 4. Their g-transformations, obtained using Eq. (16) or Table 1, are shown in Table 5. The results obtained through the application of Eqs. (14), (13) and (12) are shown in Table 6. One can see that the alternative weights obtained through the AOPA are consistent with those obtained through the OPA. It is found that the first alternative is most important with 27.1% weight. It is followed by the fourth alternative (19.3%), the third alternative (18.3%), the second alternative (17.9%) and the fifth alternative (17.3%). Thus, overall,

$$A1 > A4 > A3 > A2 > A5.$$

5. Conclusion

The study proposed the analytical (closed-form) form of the Ordinal Priority Approach (OPA), a multiple attribute decision-making methodology. Through an application it has been shown that

Table 3. The estimation of attribute weights using Analytical OPA

		C1	C2	C3	C4
Rank	E1	1	2	3	4
	E2	4	3	2	1
	E3	2	3	4	1
$\frac{1}{r_i r_{ij}}$	E1	1.000	0.500	0.333	0.250
	E2	0.125	0.167	0.250	0.500
	E3	0.167	0.111	0.083	0.333
u_j		1.292	0.778	0.667	1.083
W_j (AOPA)		0.338	0.204	0.175	0.284
W_j (OPA)		0.338	0.204	0.175	0.284

Table 4. The decision matrix containing the ranks r_{ijk}

	E1				E2				E3			
	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
A1	1	2	3	4	5	5	4	1	1	3	4	5
A2	2	3	4	5	1	4	3	3	3	2	3	3
A3	3	4	5	1	2	3	2	2	5	5	2	2
A4	4	5	1	2	3	2	1	5	4	4	1	1
A5	5	1	2	3	4	1	5	4	2	1	5	4

Table 5. The g-scores associated with r_{ijk}

	E1				E2				E3			
	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
A1	2.283	1.283	0.783	0.450	0.200	0.200	0.450	2.283	2.283	0.783	0.450	0.200
A2	1.283	0.783	0.450	0.200	2.283	0.450	0.783	0.783	0.783	1.283	0.783	0.783
A3	0.783	0.450	0.200	2.283	1.283	0.783	1.283	1.283	0.200	0.200	1.283	1.283
A4	0.450	0.200	2.283	1.283	0.783	1.283	2.283	0.200	0.450	0.450	2.283	2.283
A5	0.200	2.283	1.283	0.783	0.450	2.283	0.200	0.450	1.283	2.283	0.200	0.450

Table 6. The table containing a_{ijk} , v_k and W_k

	E1	E2	E3	v_k	W_k (AOPA)	W_k (OPA)
A1	3.299	1.313	0.572	5.183	0.271	0.271
A2	1.875	0.948	0.600	3.422	0.179	0.179
A3	1.646	1.253	0.590	3.490	0.183	0.183
A4	1.632	0.983	1.076	3.691	0.193	0.193
A5	1.965	0.712	0.634	3.311	0.173	0.173

the weights generated by the proposed Analytical Ordinal Priority Approach (AOPA) are consistent with those from the classical OPA. In future, the authors would extend the AOPA to incorporate datasets that include incompleteness and ties.

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The first author of this article serves as an Associate Editor of the journal. To ensure impartiality, the editorial and peer review process was handled independently by Dr. Hafeez Ullah (a member of the Editorial Advisory Board). The author had no involvement in the review, selection of reviewers, or editorial decision-making for this manuscript.

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