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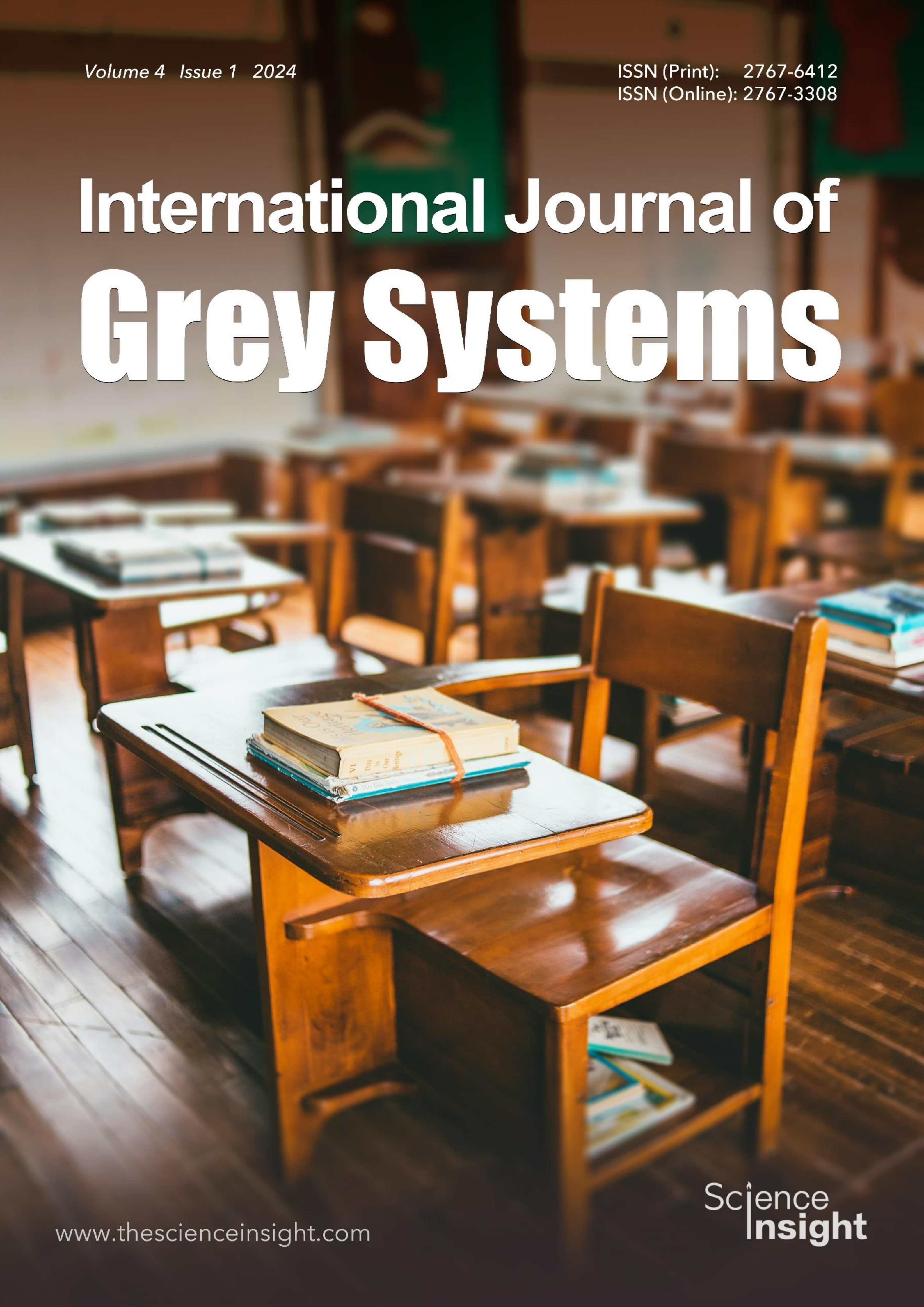

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In the memory of Professor Deng Julong (1933 - 2013),

the founder of Grey System Theory

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# Grey Multiple-Criteria Decision-Making

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**Abstract:** Decision Making (DM) is one of the most important components of human cognition. In particular, the Multiple-Criteria DM (MCDM), is a composite form of DM evaluating options with conflicting goals and choosing the best solution among the existing ones. Following the fuzzy DM criterion of Bellman and Zadeh in 1970, several other methods have been developed by other researchers for DM in fuzzy environments. Here we present a parametric, MCDM method utilizing grey numbers as tools. This method improves an earlier approach of Maji and colleagues in 2002, who used the tabular representation of a soft set as a tool for parametric MCDM in a fuzzy environment. The method is also extended to cover cases of weighted DM and suitable examples are presented illustrating our results.

**Keywords:** Grey Number; Soft Set; Tabular Representation; Decision-Making; Multiple-Criteria Decision Analysis

## 1. Introduction

Decision Making (DM), which is one of the most important components of human cognition, is the process of choosing a solution between two or more alternatives, on the purpose of achieving the optimal result for a given problem. Obviously DM has sense if, and only if, there exist more than one feasible solutions, together with one or more suitable criteria helping the decision maker to choose the best among these solutions. We recall that a solution is characterized as feasible, if it satisfies all the restrictions imposed onto the real system by the statement of the problem as well as all the natural restrictions imposed onto the problem by the real system; e.g. if $x$ denotes the quantity of the stock of a product, it must be $x \geq 0$. The choice of the suitable criterion (or criteria), especially when the results of DM are affected by random events, depends upon the desired goals of the decision maker; e.g. optimistic or conservative criteria, etc.

The rapid technological progress, the impressive development of the transportation means, the globalization of human society, the continuous changes appearing to the local and international economies, and other related reasons, led during the last 60-70 years to a continuously increasing complexity of the problems of our everyday life. As a result the DM process became in many cases a very difficult task, which is impossible to be based on the decision maker's experience, intuition and skills only, as it usually happened in the past. Thus, from the beginning of the 1950's a progressive development started of a systematic methodology for the DM process, termed as Statistical Decision Theory, which is based on principles of Probability Theory, Statistics, Economics, Psychology and other related scientific topics (Berger, 1980).

The DM process involves the following steps:

- d1: Analysis of the decision problem, i.e. understanding, simplifying and reformulating the problem in a form permitting the application of the standard DM techniques on it.
- d2: Collection and interpretation of all the necessary information related to the problem.
- d3: Determination of all the feasible solutions.
- d4: Choice of the best solution in terms of the suitable, according to the decision maker's goals, criterion (or criteria).

One could add one more step to the DM process, the verification of the chosen decision according to the results obtained by applying it in practice. However, this step is extended to areas which, due to their depth and importance, have become autonomous. Therefore, it is usually examined separately from the other steps of the DM process (e.g. see Voskoglou, 2014).

Note that the first three steps of the DM process are continuous, in the sense that the completion of each one of them usually needs some time, during which the decision maker's reasoning is characterized by transitions between hierarchically neighbouring steps. In other words, the DM process, the flow-diagram of which is represented in Figure 1, cannot be characterized as a linear process.

In particular, the Multiple-Criteria Decision Making (MCDM), is a composite form of DM evaluating options with conflicting goals and choosing the best solution (e.g. see Taherdoost & Madanchian, 2023).

DM problems appear frequently in everyday life characterized by vagueness. In such cases the classical Statistical Decision theory does not offer effective tools for studying the DM process. Fuzzy Logic (FL), on the contrary, due to its nature of including multiple values, offers a rich field of resources for this purpose. Bellman and Zadeh (1970) were the first who applied principles of FL to DM.

Following the fuzzy DM criterion of Bellman and Zadeh, several other methods were proposed by other researchers for DM in fuzzy environments; e.g. Alcantud (2018), Alazemi *et al.* (2021), Zhu and Ren (2022), Khan *et al.* (2022), Chiclana *et al.* (1998), Ekel (2001, 2002), Ekel *et al.* (2016), etc. Here we will develop a parametric, MCDM method using soft sets (SSs), and grey numbers (GNs) as tools (Voskoglou, 2023a). This method improves an earlier method of Maji *et al.* (2002), which uses the tabular representation of a SS as a tool for MCDM in a fuzzy environment.

The present study is formulated as follows: Section 2 contains the necessary mathematical background about GNs and SSs, as well as the description, through a suitable example, of the parametric MCDM method of Maji *et al.* (2002). Section 3, after pointing out the weaknesses of the previous method, presents the improved method using GNs as tools, which results in better decisions than the method of Maji *et al.* when at least one of the parameters involved has a fuzzy texture. Section 4 discusses the weighted MCDM and the study closes with Section 5 including the final conclusions and some hints for future research.

## 2. Mathematical Background

### *2.1 Grey Numbers*

The grey system (GS) theory was introduced by Deng (1982) for handling approximate data (Liu & Lin, 2010). The main tool for handling the approximate data of a GS is the use of GNs. Modern readers often find GS theory as a nonparametric alternative to fuzzy set theory (Zadeh, 1965) because of its flexible propositions and arithmetic.

d1 ↔ d2 ↔ d3 → d4

**Fig 1.** The flow-diagram of the decision-making process

DEFINITION 1: A GN $L$, denoted with $\otimes L$ is understood to be a real number with known range given by a closed real interval of the form $[a, b]$, but with unknown exact value. The GN $\otimes L$, however, may differ from the interval $[a, b]$ with respect to the presence of a whitenization function $f: [a, b] \rightarrow [0,1]$, such that the closer is $f(t)$ to 1, the better $t \in [a, b]$ approximates the unknown value of $\otimes L$.

When no such function exists, it is logical to consider as the crisp representative of $L$ the real number

$$\mathrm{K}(\otimes L) = \frac{a+b}{2} \tag{1}$$

The real number $\mathrm{K}(\otimes L)$ is usually referred to as the kernel of $\otimes L$ and the process of calculating its kernel is usually referred to as the whitenization of $\otimes L$.

The known arithmetic of the real intervals (Moore et al, 1995) is used to perform the basic arithmetic operations between GNs. Let $\otimes L_1 \in [a_1, b_1]$ and $\otimes L_2 \in [a_2, b_2]$ be given GNs and let r be a positive number. In this paper we will make use only of the addition and of the scalar product of GNs, which are defined respectively by the relations

$$\otimes L_1 + \otimes L_2 \in [a_1 + b_1, a_2 + b_2] \tag{2}$$

and

$$r \otimes L_1 \in [ra_1, rb_1]. \tag{3}$$

### *2.2 Using Soft Sets for Parametric Decision Making*

Molodtsov (1999) introduced the notion of SS for a parametric treatment of the real-world uncertainty in the following way:

DEFINITION 2: Let $E$ be a set of parameters and let $f$ be a map from $E$ into the power set $P(U)$ of the universal set $U$. Then the SS $(f, E)$ in $U$ is defined as a parameterized family of subsets of $U$ by

$$(f, E) = \{(e, f(e)): e \in A\} \tag{4}$$

The term "soft" was introduced because the form of $(f, E)$ depends on the parameters of $E$.

Maji *et al.* (2002) introduced the *tabular representation* of a SS for storing it easily in a computer's memory and they used it for parametric DM. The following example illustrates their DM methodology.

EXAMPLE 1: A company wants to employ a person among six candidates, say $A_1$, $A_2$, $A_3$, $A_4$, $A_5$ and $A_6$. The ideal qualifications for the new employee is to have satisfactory previous experience, to hold a university degree, to have a driving license and to be young. Assume that $A_1$, $A_2$, $A_6$ are the candidates with satisfactory previous experience, $A_2$, $A_3$, $A_5$, $A_6$ are the holders of a university degree, $A_3$, $A_5$ are the holders of a driving license and $A_4$ is the unique young candidate. Find the best decision for the company.

SOLUTION: Set $U = \{A_1, A_2, A_3, A_4, A_5, A_6\}$ and let $F = \{p_1, p_2, p_3, p_4\}$ be the set of the parameters $p_1$=well experienced, $p_2$=holder of a university degree, $p_3$=holder of a driving license and $p_4$=young. Consider the map $f: F \rightarrow P(U)$ defined by

$f(p1) = \{A_1, A_2, A_6\}, f(p2) = \{A_2, A_3, A_5, A_6\}, f(p3) = \{A_3, A_5\}, f(p4) = \{A_4\}$.

Then the SS defined with respect to $F$ and $f$ is equal to

$$(f, F) = \{(p1, \{A_1, A_2, A_6\}), (p2, \{A_2, A_3, A_5, A_6\}), (p3, \{A_3, A_5\}), (p4, \{A_4\})\}$$

The tabular representation of the SS $(f, F)$, shown in Table 1, is formed by assigning to each candidate the binary element 1, if he/she satisfies the qualification addressed by the corresponding parameter, or the binary element 0 otherwise.

Then, the *choice value* $V$ of each candidate is determined by adding the entries of the row of the tabular representation of $(f, F)$ where it belongs. Thus, the candidates $A_1$ and $A_4$ have choice

**Table 1.** Tabular representation of the SS (f, F) of Example 1

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ |
|---|---|---|---|---|
| $A_1$ | 1 | 0 | 0 | 0 |
| $A_2$ | 1 | 1 | 0 | 0 |
| $A_3$ | 0 | 1 | 1 | 0 |
| $A_4$ | 0 | 0 | 0 | 1 |
| $A_5$ | 0 | 1 | 1 | 0 |
| $A_6$ | 1 | 1 | 0 | 0 |

value 1 and all the others 2. The company, therefore must employ one of the candidates $A_2$, $A_3$, $A_5$ or, $A_6$.

## 3. Grey Decision-Making

The DM method of Maji al. is not very helpful for the company in Example 1 to choose the new employee, since it excluded only two (A1 and A4) among the six candidates. This is due to the fact that in the tabular matrix of the corresponding SS the characterization of the candidates by the corresponding parameters was done by using the binary elements (truth values) 0, 1. In other words, although the method of Maji and colleagues starts from a fuzzy basis utilizing SSs as tools, then it uses bivalent logic for making the required decision. This could lead to inadequate decisions, if some of the parameters have a fuzzy texture, like it happens with the parameters $p_1$: well-experienced and $p_4$: young of Example 1. For tackling this problem, we have used GNs instead of the binary elements 0, 1 in the tabular representation of the corresponding SS (Voskoglou, 2023a). This methodology is illustrated here with the following example:

EXAMPLE 2: Revisit Example 1 and assume that the analysts of the company, after studying more carefully the available information for the six candidates, decided to use the GNs $G_1 \in [0.85, 1]$, $G_2 \in [0.75, 0.84]$, $G_3 \in [0.6, 0.74]$, $G_4 \in [0.5, 0.59]$ and $G_5 \in [0, 0.49]$ instead of the binary elements 0, 1 for characterizing the parameters $p_1$ and $p_4$, which have a fuzzy texture, as shown in Table 2. Which is the best decision for the company in this case?

SOLUTION: Adopting the notation used in the solution of Example 1, Table 2 gives the revised tabular representation of the SS $(f, F)$. In this case the choice values are calculated through the whitenization of the GNs. Consequently, with the help of formulas (1) and (2) one finds that

$$V_1 = 0 + 0 + K(G_1 + G_3) = K([0.85 + 0.6, 1 + 0.74]) = \frac{1.45 + 1.74}{2} = 1.595$$

and similarly,

$$V_2 = 1 + 0 + K(G_1 + G_5) = 2.17,$$
$$V_3 = 1 + 1 + K(G_3 + G_3) = 3.34,$$
$$V_4 = 0 + 0 + K(G_4 + G_1) = 1.47,$$
$$V_5 = 1 + 1 + K(G_4 + G_3) = 3.215,$$
$$V_6 = 1 + 0 + K(G1 + G4) = 2.47.$$

**Table 2.** Characterizations of the parameters involved in Example 2

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ |
|---|---|---|---|---|
| $A_1$ | $\otimes G_1$ | 0 | 0 | $\otimes G_3$ |
| $A_2$ | $\otimes G_1$ | 1 | 0 | $\otimes G_5$ |
| $A_3$ | $\otimes G_3$ | 1 | 1 | $\otimes G_3$ |
| $A_4$ | $\otimes G_4$ | 0 | 0 | $\otimes G_1$ |
| $A_5$ | $\otimes G_4$ | 1 | 1 | $\otimes G_3$ |
| $A_6$ | $\otimes G_1$ | 1 | 0 | $\otimes G_4$ |

Therefore, the best decision for the company is to choose the candidate $A_3$. This is obviously a better decision than that the one made in Example 1, because it leads to the choice of only one candidate ($A_3$) not creating dilemma for the company like it happened in the case presented in Example 1 (choice of one among the candidates $A_2$, $A_3$, $A_5$ and $A_6$).

## 4. Weighted Decision-Making

DM cases appear frequently in everyday life in which the decision maker's goals are not equally important. In such cases, weight coefficients, whose sum is equal to 1, are assigned to each parameter. This is illustrated here with the following example.

EXAMPLE 3: Revisit Examples 1 and 2 and assume that the weight coefficients 0.4, 0.3, 0.2 and 0.1 have been assigned to the parameters $p_1$, $p_2$, $p_3$ and $p_4$ respectively according to the importance of the goals of the company. Which is the best choice for the company under these conditions?

SOLUTION: In case of Example 1, after incorporating the weights, the new choice values of the candidates become 0.4 ($A_1$), 0.7 ($A_2$), 0.5 ($A_3$), 0.1 ($A_4$), 0.5 ($A_5$) and 0.7 ($A_6$). Therefore, the company must choose one of the candidates $A_2$ or $A_6$.

In case of Example 2, with the help of formulas (1), (2) and (3) one finds that the weighted choice value of the candidate A1 is equal to

$$V1 = K[0.4(G1) + 0.1(G3)] = 0.437.$$

Similarly

$$V2 = 0.3 + K[0.4(G1) + 0.1(G5)] = 0.6945,$$
$$V3 = 0.3 + 0.2 + K[0.4(G3) + 0.1(G3)] = 0.835,$$
$$V4 = K[0.4(G4) + 0.1(G1)] = 0.168,$$
$$V5 = 0.3 + 0.2 + K[0.4(G4) + 0.1(G3)] = 0.758,$$
$$V6 = 0.3 + K[0.4(G3) + 0.1(G4)] = 0.6225.$$

Therefore, the best decision for the company is to employ the candidate $A_3$.

In conclusion, if the analysts of the company are sure about the qualifications of the six candidates, following the weighted DM method of Maji *et al.* of the revised Example 1, must choose one of the candidates $A_2$ or $A_6$. Otherwise, following the weighted grey DM method of the revised Example 2 (i.e. using GNs in the decision matrix instead of the binary elements 0, 1) must choose the candidate $A_3$.

## 5. Discussion and conclusion

Following the introduction of the theory of fuzzy sets by Zadeh (1965), various extensions and related theories have been developed through the years for a more effective management of the existing in the real-world uncertainty; e.g. see Voskoglou (2019a). Each one of them is suitable for tackling one or more types of uncertainty, but none of them can tackle alone all the existing forms of it. All these theories together, however, form an adequate framework for managing the uncertainty in general.

Furthermore, suitable combinations of the previous theories seem to provide better results. In this work, for example, using a combination of SSs and GNs, we developed a hybrid model for parametric MCDM, which improves an earlier model of Maji *et al.* (2002) using only SSs as tools. Similar MCDM models were developed by the present author, in which the binary elements 0, 1 in the tabular matrix of the corresponding SS were replaced either by intuitionistic fuzzy pairs (Voskoglou, 2023b), or by neutrosophic triplets (Voskoglou, 2023c), depending on the form of the corresponding DM problem. Taking into account that analogous hybrid models were also developed for the assessment of several human or machine activities under fuzzy conditions (e.g. see Voskoglou, 2019a), one concludes that this is an interesting and much promising approach for further research.

## Originality statement

The author(s) declares that the work reported in the current study is original, and no content (concept, text, tables, illustrations, data, etc.) supposed to be produced/generated/estimated/written/collected by the author(s) in the current study is partially or completely generated through Artificial Intelligence (AI) or any AI-based software.

## References

Alazemi, F. K. A., Ariffin, M. K. A. B. M., Mustapha, F. B., & Supeni, E. E. B. (2021). A comprehensive fuzzy decision-making method for minimizing completion time in manufacturing process in supply chains. *Mathematics*, *9*(22), 2919. https://doi.org/10.3390/math9222919

Alcantud, J. C. R. (2018). Fuzzy techniques for decision making. *Symmetry*, *10*(1), 6. https://doi.org/10.3390/sym10010006

Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management Science*, *17*(4), B-141. https://doi.org/10.1287/mnsc.17.4.B141

Berger J. O. (1980). *Statistical Decision Theory: Foundations, Concepts and Methods*. New York: Springer-Verlag. https://doi.org/10.1007/978-1-4757-1727-3

Chiclana, F., Herrera, F., & Herrera-Viedma, E. (1998). Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations. *Fuzzy Sets and Systems*, *97*(1), 33-48. https://doi.org/10.1016/S0165-0114(96)00339-9

Deng, J. (1982). Control problems of grey systems. *Systems & Control Letters*, *1*(5), 288-294. https://doi.org/10.1016/S0167-6911(82)80025-X

Ekel, P. Y. (2001). Methods of decision making in fuzzy environment and their applications. *Nonlinear Analysis: Theory, Methods & Applications*, *47*(2), 979-990. https://doi.org/10.1016/S0362-546X(01)00239-5

Ekel, P. Y. (2002). Fuzzy sets and models of decision making. *Computers & Mathematics with Applications*, *44*(7), 863-875. https://doi.org/10.1016/S0898-1221(02)00199-2

Ekel, P., Kokshenev, I., Parreiras, R., Pedrycz, W., & Pereira Jr, J. (2016). Multiobjective and multiattribute decision making in a fuzzy environment and their power engineering applications. *Information Sciences*, *361*, 100-119. https://doi.org/10.1016/j.ins.2016.04.030

Khan, A., Yang, M. S., Haq, M., Shah, A. A., & Arif, M. (2022). A new approach for normal parameter reduction using σ-algebraic soft sets and its application in multi-attribute decision making. *Mathematics*, *10*(8), 1297. https://doi.org/10.3390/math10081297

Liu, S., & Lin, Y. (Eds.). (2010). *Advances in Grey System Research*. Berlin – Heidelberg, Germany: Springer. https://doi.org/10.1007/978-3-642-13938-3

Maji, P. K., Roy, A. R., & Biswas, R. (2002). An application of soft sets in a decision making problem. *Computers & Mathematics with Applications*, *44*(8-9), 1077-1083.

Molodtsov, D. (1999). Soft set theory – first results. *Computers and Mathematics with Applications*, *37*(4-5), 19-31. https://doi.org/10.1016/S0898-1221(99)00056-5

Moore, R.A., Kearfort, R. B., & Clood, M.J. (1995). *Introduction to Interval Analysis* (2nd Printing). Philadelphia, USA: SIAM.

Taherdoost, H., & Madanchian, M. (2023). Multi-criteria decision making (MCDM) methods and concepts. *Encyclopedia*, *3*(1), 77-87. https://doi.org/10.3390/encyclopedia3010006

Voskoglou, M.Gr. (2014). Probability and Fuzziness in Decision Making. *Egyptian Computer Science Journal*, *38*(3), 86-99. http://www.ecsjournal.org/Archive/Volume38/Issue3/9.pdf

Voskoglou, M.Gr. (2019a). *Generalizations of Fuzzy Sets and Related Theories*, in M. Voskoglou (Ed.), An Essential Guide to Fuzzy Systems, Commentary, 345-352, Nova Publishers, N.Y.

Voskoglou, M.Gr. (2019b). Methods for Assessing Human-Machine Performance under Fuzzy Conditions. *Mathematics*, *7*, 230. https://doi.org/10.3390/math7030230

Voskoglou, M.Gr. (2023a). A Combined Use of Soft Sets and Grey Numbers in Decision Making. *Journal of Computational and Cognitive Engineering*, *2*(1), 1-4. https://doi.org/10.47852/bonviewJCCE2202237

Voskoglou, M.Gr. (2023b). Applications of Intuitionistic Fuzzy Sets to Assessment, and Decision Making. *Journal of Fuzzy Extensions and Applications*, *4*(4), 299-309. https://doi.org/10.59400/jam.v1i4.325

Voskoglou, M.Gr. (2023c). An Application of Neutrosophic Sets to Decision Making. *Neutrosophic Sets and Systems*, *53*, 1-9.

Zadeh, LA. (1965). Fuzzy Sets. *Information and Control*, *8*, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zhu, B. & Ren, P. (2022). Type-2 fuzzy numbers made simple in decision making. *Fuzzy Optimization and Decision Making*, *21*, 175-195. https://doi.org/10.1007/s10700-021-09363-y

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# A Novel Grey Fuzzy Clustering Model and its Application on Students' Performance Evaluation

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**Abstract:** Aiming to address the issues of traditional student evaluation methods, which tend to be overly subjective and overlook the intrinsic data structure, this paper introduces a novel grey fuzzy clustering model named the grey entropy game-weighted fuzzy c-means (GEG-WFCM) model. Firstly, subjective weights are calculated using the subjective-objective relationship analysis method, while objective weights are determined through the entropy weight method. Then, a comprehensive approach is adopted, leveraging game theory to calculate the final weights. Based on these comprehensive weights, the relative grey correlation coefficient and fuzzy weighted c-mean algorithm are incorporated to yield the ultimate evaluation results. The proposed model was applied to evaluate student performance, and the experiments show that it can obtain scientific and reasonable results. The model not only acknowledges the expertise of experts but also respects the objectivity of the data, thus circumventing the limitations of purely subjective judgments, and surpassing traditional evaluation methods.

**Keywords:** Grey Correlation; Comprehensive Evaluation; Entropy Weight; Clustering; Fuzzy C-Means

## 1. Introduction

Evaluation of student performance is a comprehensive assessment of the learning process and certain stages of learning, which is designed to enable students to understand their mastery of knowledge and to identify their progress and shortcomings at this stage (Wu *et al.*, 2017). A scientific comprehensive student evaluation system can objectively reflect the problems in the teaching process and play a positive role in guiding teachers' teaching and students' development. Therefore, the establishment of a scientific and effective comprehensive evaluation system for students is a problem that needs to be solved urgently.

Numerous scholars have put forward various evaluation algorithms. For example, Liu *et al.* (2012) used AHP to identify the evaluation objectives and index weights, and fuzzy comprehensive evaluation to comprehensively evaluate the students' vocational ability. Wu *et al.* (2017) used a combination of factor analysis and cluster analysis to evaluate students' performance and finally classified students into several clusters through cluster analysis based on factor scores for objective and comprehensive evaluation. Rosadi *et al.* (2017) grouped data by fuzzy C-mean clustering algorithm when assessing students' academic performance and validated their application. Wang (2022) improved the K-means algorithm based on student information to address the problem of large course differences in the evaluation of student performance and proved that the improved pan of this paper has obvious advantages through data. Ren *et al.* (2022) proposed a compartment energy consumption analysis model based on combined weighting (AHP and entropy weighting method) and grey-fuzzy comprehensive evaluation, which comprehensively evaluated the energy consumption of 22 compartments under the same special painting process, and selected the most reasonable cabin energy consumption scheme. Luo *et al.* (2022) propose a health condition rating system covering system level and device level, use the G1 group method and projection recourse method to get the optimal weights, propose an improved grey TOPSIS comprehensive evaluation model based on prospect theory, and verify the validity of the proposed model in this paper. Kou *et al.* (2022) determined the combination weights of the factors based on game theory, established a comprehensive evaluation model of the open pit mine truck scheduling system based on grey correlation analysis (GRA-TOPTSIS), and verified the effectiveness of the model. Gu *et al.* (2023) proposed an employment quality evaluation model based on grey correlation and fuzzy C-mean (FCM) to address the defects of large errors in employment quality evaluation and compared the algorithm with other employment quality evaluation models to verify the superiority of the model. Zhao (2023) establishes a health state estimation method that combines grey clustering and fuzzy comprehensive evaluation methods to evaluate the health state of the power supply under multiple sets of data using grey clustering and demonstrates through examples that the method is effective in estimating the operating condition of the power supply that is normally degraded.

Most evaluation models are mainly constructed with fuzzy comprehensive evaluation, cluster analysis, grey cluster analysis, and other evaluation models combined with single-weight assignment methods. Nevertheless, the single-weight assignment way can't fully consider the characteristics of different indicators and the relationship between them, leading to unreasonable weight assignment. Fuzzy comprehensive evaluation relies on the subjective judgment of experts' experience, which may lead to insufficient accuracy of the evaluation results. A single clustering method may not be able to accurately reveal the intrinsic structure of data when dealing with complex and high-dimensional data.

In this study, we construct a fuzzy c-mean weighted clustering evaluation model, named GEG-WFCM, for comprehensive evaluation. The main contributions are:

1. By integrating the G1 method and entropy weight method through game theory, the model can comprehensively and flexibly consider both subjective and objective factors, leading to a more reasonable allocation of weights.
2. By introducing the relative grey correlation coefficient, the model objectively reflects the relative relationship between evaluation objects and evaluation indices, thus reducing the interference of subjective factors and improving the accuracy of the data.
3. By combining the relative gray correlation coefficient with the weighted fuzzy c-means clustering algorithm (WFCM), the model can better handle fuzzy and uncertain data, improving the accuracy and stability of clustering results.
4. The model is applied to the student comprehensive performance evaluation, and the results verify the effectiveness of the model.

## 2. GEG-WFCM Model

### *2.1 Preliminaries*

*2.1.1 The group G1 method (G1).* The G1 method, also known as the ordinal relationship analysis method, is a subjective weighting method that is an improvement of the Analytic Hierarchy Process (AHP). This method uses the ordinal relationship established by decision-makers between different indicators as a standard for assigning weights to indicators (Ye *et al.*, 2023).

*2.1.2 The entropy weight method (EWM).* The EWM was developed from the information entropy theory proposed by Claude Shannon (Shannon, 1948), and it is an objective assignment method for determining weights, which is widely used in comprehensive evaluations of multiple indicators and decision analysis. The method is based on the concept of information entropy and determines the weight of each indicator by measuring its information.

*2.1.3 The game theory (GT).* The theoretical foundation of game theory can be traced back to the classic book published by John von Neumann and Oskar Morgenstern (von Neumann & Morgenstern, 1947). Game theory mainly studies the interaction between formularized incentive structures. It is a mathematical theory and method for studying phenomena with a struggle or competitive nature. Game theory considers the predicted and actual behaviors of individuals in a game and studies their optimal strategies.

*2.1.4 The Grey Relational Coefficient (GRC).* The GRC is used to measure the degree of correlation between two sequences and is suitable for decision analysis in the case of incomplete information. The grey correlation coefficient stresses the relative changes between the series and better reflects the actual correlation between the factors (Deng, 1989).

*2.1.5 The weighted fuzzy C-means (WFCM).* The FCM is a classical clustering algorithm that describes the degree to which each data point belongs to each cluster by dividing it into multiple clusters and assigning an affiliation to each data point (Peizhuang, 1983). Later, some scholars introduced weights to improve the FCM algorithm, i.e., weighted fuzzy C-mean (WFCM), which can reflect the importance of the indicators well enough and improve the accuracy of the clustering results. It can also adaptively adjust the weights of each feature to provide more flexible clustering results with better adaptability.

The current study firstly obtains subjective weights by the G1 method, objective weights by the entropy weight method, and finally comprehensive weights by game theory, thus overcoming the disadvantage of too much subjectivity of the AHP method. Then, the relative grey correlation coefficient of each index of each evaluation object is calculated, and the coefficient is brought into the WFCM clustering algorithm to get the final evaluation result. This evaluation method fully takes into account the non-linear relationship between the data, thus making the evaluation results more adaptable and objective.

### *2.2 Combined subjective and objective weighting solution*

*2.2.1 Initialize the original matrix.* Assuming $n$ students and $P$ indicators, let $A = \{A_1, A_2, A_3, \ldots, A_n\}$ be the set of students and $C = \{C_1, C_2, C_3, \ldots, C_p\}$ be the set of indicators to obtain a raw data matrix $D$ of $N * P$ :

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & \cdots & X_p \end{array} \\ \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1p} \\ E_{21} & E_{22} & \ldots & E_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{np} \end{bmatrix} \end{array} \tag{1}$$

*2.2.2 Calculate the indicator subjective weight.* The G1 method is in essence a subjective weighting method improved and optimised based on hierarchical analysis. When the AHP has a large amount of data, too many evaluation indicators, and is too cumbersome to solve, the G1 method overcomes these shortcomings by introducing the idea of group decision-making and considering the weight of each expert.

Each expert's weight is $d_q$ and $0 < d_q < 1$.

$$W_j = \sum_{i=1}^{m} d_q \omega_j^q \tag{2}$$

where $W_j$ denotes the composite weight of the indicator at $j$, $\omega_j^q$ is the weight of the indicator at $q$ for the expert and $m$ is the total number of experts. Thus, the subjective weights are obtained.

*2.2.3 Determine the objective weight of each indicator using the entropy weighting method.* The data in D was first normalized according to the following formula:

$$e_{ij} = \frac{E_{ij} - \min(E_{1j}, E_{nj})}{\max(E_{1j}, E_{nj}) - \min(E_{1j}, E_{nj})} \quad (1 \le i \le n, 1 \le j \le p) \tag{3}$$

Firstly, based on the standardized decision matrix, the weight of each evaluation object under each indicator is obtained, i.e. the weight of the $i^{\text{th}}$ evaluation object concerning the value of the $j^{\text{th}}$ indicator.

$$P_{ij} = \frac{e_{ij}}{\sum_i^n e_{ij}}, 0 \le P_{ij} \le 1 \tag{4}$$

It is then possible to create the weighting matrix for the new data as follows:

$$P = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{matrix} \begin{matrix} \begin{matrix} X_1 & X_2 & \cdots & X_p \end{matrix} \\ \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1p} \\ p_{21} & p_{22} & \cdots & p_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{np} \end{bmatrix} \end{matrix} \tag{5}$$

Then find the entropy value of each indicator, according to equation (6):

$$e_j = -\frac{1}{\ln(n)} \sum_{i=1}^{n} p_{ij} \ln p_{ij}, \, j = 1,2,\ldots, e_j \ge 0 \tag{6}$$

The entropy method assigns weights based on the degree of difference in the sign values of each indicator, which results in the corresponding weights of each indicator:

$$d_j = 1 - e_j \tag{7}$$

If $p$ denotes the number of indicators then the weight of each indicator is defined as

$$W_j = \frac{d_j}{\sum_{i=1}^{p} d_i} \tag{8}$$

Finally, the objective weights of indicators are obtained.

*2.2.4 Solve for composite weights using the game theory methodology approach.* According to game theory, $N$ weight calculation methods are chosen to form a set of weights $W = \{w_1, w_2, w_3, \ldots, w_n\}$, for any linear combination of these $N$ vectors, $w = \sum_{k=1}^{N} a_k w_k^T$ that can be obtained, and dispersion minimization is performed for $w$ and $w_k$ according to the optimal strategy.

$$Min \| \sum_{k=1}^{N} a_k w_K^T - w_j^T \|^2, j = (1,2,\cdots,N). \tag{9}$$

Based on the differential properties of matrices, the optimal first-order derivative condition satisfies the following equation:

$$\begin{bmatrix} w_1 w_1^T & \cdots & w_1 w_N^T \\ \vdots & & \vdots \\ w_N w_1^T & \cdots & w_N w_N^T \end{bmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix} = \begin{bmatrix} w_1 w_1^T \\ \vdots \\ w_N w_N^T \end{bmatrix} \tag{10}$$

The coefficients $(a_1, a_2, a_3, \ldots, a_n)$ were normalized to obtain the optimal weighting coefficients and the final weights were determined as follows:

$$w^* = \sum_{k=1}^{N} a_k^* w_k^T \tag{11}$$

## *2.3 Evaluation result solving*

*2.3.1 Data standardisation.* In order to carry out the data into the FCM algorithm for cluster analysis, the raw data is first normalized with the following formula:

$$h_{ij} = \frac{E_{ij}}{\max(E_{1j}, \ E_{nj})} (1 \le i \le n, 1 \le j \le p) \tag{12}$$

The normalization matrix obtained is:

$$H = \begin{matrix} & X_1 & X_2 & \cdots & X_p \\ A_1 & h_{11} & \mathrm{h}_{12} & \cdots & \mathrm{h}_{1p} \\ A_2 & h_{21} & \mathrm{h}_{22} & \cdots & \mathrm{h}_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_n & h_{n1} & h_{n2} & \cdots & h_{np} \end{matrix} \tag{13}$$

*2.3.2 Calculate the grey correlation coefficient.* The optimal sequence $A^+$ and the worst sequence $A^-$ were obtained from the normalized evaluation matrix, and the grey correlation coefficients between each index and the positive ideal solution for each evaluation object were calculated according to Equation (14):

$$\gamma_{\mathrm{ij}}^+ = \gamma(A_j^+, A_{\mathrm{ij}}) = \frac{\min_i \min_j S(A_j^+, A_{ij}) + \rho \max_i \max_j S(A_j^+, A_{ij})}{S(A_j^+, A_{ij}) + \rho \max_i \max_j S(A_j^+, A_{ij})} (1 \le i \le n, 1 \le j \le p) \tag{14}$$

where $\rho \in [0,1]$, in general, and $\rho = 0.5$ are substituted to find the result.

The grey correlation coefficient of each indicator with the negative ideal solution was calculated according to equation (15):

$$\gamma_{\mathrm{ij}}^- = \gamma(A_j^-, A_{ij}) = \frac{\min_i \min_j S(A_j^-, A_{ij}) + \rho \max_i \max_j S(A_j^-, A_{ij})}{S(A_j^-, A_{ij}) + \rho \max_i \max_j S(A_j^-, A_{ij})} (1 \le i \le \mathrm{n}, 1 \le j \le p) \tag{15}$$

*2.3.3 Calculate the relative grey correlation coefficient.* According to the grey correlation coefficient of each index of the evaluation object with the positive and negative ideal solutions can be obtained as the relative grey correlation coefficient, the formula is as follows:

$$\varepsilon_{ij} = \frac{\gamma_{ij}^{+}}{\gamma_{ij}^{+} + \gamma_{ij}^{-}} (1 \le i \le n, 1 \le j \le p) \tag{16}$$

This results in a new decision matrix as follows:

$$R = \begin{matrix} & X_1 & X_2 & \cdots & X_p \\ A_1 & \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1p} \\ A_2 & \varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_n & \varepsilon_{n1} & \varepsilon_{n2} & \cdots & \varepsilon_{np} \end{matrix} \tag{17}$$

*2.3.4 Put the new grey correlation coefficient decision matrix into the weighted FCM cluster analysis.* Regarding the evaluation sample set $A = \{A_1, A_2, A_3, \dots, A_n\}$ , $n$ represents the number of samples; $k$ evaluation samples are: $A = \{A_1, A_2, A_3, \dots, A_{kp}\}$, $p$ represents the number of indicators. The quality of the evaluation is mainly divided into $C$ levels, i.e. there are $C$ clustering centers, which can be represented as $V = \{V^1, V^2, \dots, V^C\}$, then the membership matrix can be represented as $U = \{u_{ik}\}_{C \times V}$ , which meets the following conditions

$$\sum_{i=1}^{c} u_{ik} = 1, 1 \le k \le n \tag{18}$$

where $u_{ik} \in [0,1], 1 \le i \le C, 1 \le k \le n,$ denotes the degree of affiliation belonging to the $k$ evaluation sample at the $i$ level.

$$d_{ik}^{(w)} = \sum_{j=1}^{p} w_j \| A_{kj} - v_{ij} \| \tag{19}$$

$d_{ik}^{(w)}$ is the weighted Euclidean distance with indicator weights, $w_j$ is the weight of the $j^{\text{th}}$ indicator, and the objective function is defined as:

$$J(U,V) = \sum_{i=1}^{c} \sum_{k=1}^{n} (u_{ik})^m (d_{ik}^{(w)}) \tag{20}$$

where $m$ is the fuzzy weighted index.

*2.3.5 Update the sample cluster centers to get the final clustering results.* The grade of the evaluation quality result is $C$, the initialized clustering center for quality assessment is $V_0$, the number of iterations is $t = 0$ and the iteration stopping threshold is set to $\varepsilon$. The membership of the evaluation sample is calculated according to the formula.

$$u_{ik} = \sum_{j=1}^{c} (d_{ik}^{(w)} / d_{jk}^{(w)})^{\frac{2}{m-1}} \tag{21}$$

Update the cluster center as follows:

$$v_i = \sum_{j=1}^{n} u_{ik}^m x_j / \sum_{j=1}^{n} u_{ik}^m \tag{22}$$

If $\mid V_{(k+1)} - V_k \mid < \varepsilon$ , the clustering center is obtained, and the evaluation model is built according to the clustering center, otherwise, $k = k + 1$ returns to the calculation iteration, i.e., Equation (22), and the final iteration stops, the clustering center of each evaluation object is obtained, and according to the principle of the maximum degree of affiliation, the clustering result of the evaluation object is obtained.

## 3. Students' performance evaluation with the GEG-FCM model

In this section, we take the comprehensive evaluation of student's performance in the course of "Programming Language" at a university as an example, select two classes with a total of 94 students' course performance as the experimental data samples, and obtain the clustering results and analyses and discusses them by bringing the sample data into the algorithmic procedure. All data are analyzed and calculated with Python software.

*3.1 Students' performance evaluation*

*3.1.1 Establishment of evaluation indicators.* Based on the evaluation of technical courses in higher education and the characteristics of student evaluation, combined with the principles of developmental and diagnostic evaluation, specific student course evaluations are designed with the following indicators:

(1) Classroom performance: students' performance in participating in the classroom learning process, including attendance, class discussion, hands-on work, group inquiry, answering questions, etc.

(2) Assignment grades: including online and offline assignments and experimental designs

(3) Curriculum Design: Comprehensive Major Assignment Design and Report Presentation

(4) Mid-term test: a stage test of students' learning and application of course knowledge in the first half of the semester.

(5) Final Examination: A summative examination of the student's learning and application of course knowledge during the semester.

The specific system for constructing indicators is shown in Table 1.

*3.1.2 Determination of subjective weights of indicators.* According to the calculation steps of the G1 method, let the experts rank and score the importance of each indicator, and finally get the subjective weight of each indicator obtained by the G1 method $W_1 = [0.0955, 0.1337, 0.1873, 0.24305, 0.3402]$, the scoring results are displayed in Table 2.

*3.1.3 Determination of objective weights for indicators.* The original data matrix is normalized and the objective weights of each indicator are finally obtained through the calculation steps of each formula of the entropy weighting method: $W_2 = [0.0895, 0.06673, 0.5313, 0.18482, 0.1274]$.

*3.1.4 Determination of composite weights for indicators.* According to the calculation method of game theory and the relevant formula, the two methods are combined with the optimal strategy to obtain the weight coefficients of 0.248 and 0.752, respectively. Then the final coupling weights are synthesised as $W = [0.091, 0.083, 0.446, 0.199, 0.18]$.

*3.1.5 Data standardisation.* The standardization of the students' scores on the various indicators resulted in the raw data matrix, as shown in Table 2.

**Table 1.** Indicator system for comprehensive evaluation of student achievement

| *Level 1 indicators* | Secondary indicators |
|---|---|
| *Ordinary grades* | Classroom performance |
| | Assignment grades |
| | Curriculum design |
| *Examination results* | Mid-Term test |
| | Final examination |

**Table 2.** Raw data on student performance

| *Student number* | *Classroom performance* | *Assignment grades* | *Curriculum design* | *Mid-term test* | *Final examination* |
|---|---|---|---|---|---|
| 1 | 0.99 | 0.99 | 0.9789 | 0.5111 | 0.4081 |
| 2 | 0.98 | 1 | 0.9889 | 0.9111 | 0.7040 |
| 3 | 0.97 | 1 | 0.9684 | 0.9 | 0.5612 |
| 4 | 0.95 | 0.94 | 0.9789 | 0.9222 | 0.5408 |
| 5 | 0.94 | 1 | 0.9684 | 0.8555 | 0.8979 |
| 6 | 0.88 | 1 | 0.9789 | 0.8777 | 0.5816 |
| … | … | … | … | … | … |
| 94 | 1 | 1 | 0.9684 | 0.7111 | 0.4081 |

*3.1.6 Determining positive and negative ideal solutions.* Based on the scoring matrix, the maximum value of each indicator is selected as the ideal solution, and the minimum value of each indicator is selected as the negative ideal solution:

$$A^{+} = [1.0, 1.0, 1.0, 1.0, 1.0]$$

$$A^{-} = [0.85, 0.70, 0.94736842, 0.477777, 0.0408163]$$

*3.1.7 Calculation of the relative grey correlation coefficients.* Finally, the relative grey correlation coefficients for each indicator for each student were obtained according to the formula, and the coefficients are listed in Table 3.

*3.1.8 FCM weighted cluster analysis.* The above grey correlation coefficient matrix, brought to the design of the FCM weighted clustering algorithm program, sets the number of clusters for 4, the student performance is divided into four categories, respectively, the results are unsatisfactory, the results are average, good results, good results of the four major categories, respectively, with 1 ~ 4 to correspond to the various levels. After many iterations, the final weighted clustering results are obtained, as shown in Figure 1.

The normalized data is brought into the weighted FCM algorithm for calculation and the results are compared and analyzed with the grey FCM algorithm as shown in Figure 2.

It can be seen from Figure 2 that the two curves converge, indicating that the two algorithms are roughly the same in terms of clustering results, and the agreement rate is about 95.7%, which can be concluded that the grey FCM clustering algorithm is more scientific and reasonable. The results of the final student evaluation are shown in Table 4.

*3.2 Results*

According to the data in Figure 1, it can be found that most of the student's grades are at the average level and above, and only a small number of students' grades are at the unsatisfactory level, which indicates that most of the students in the course meet the overall objectives of the course and have a good learning effect. Among the students whose course grades are at an unsatisfactory

**Table 3.** Relative grey correlation coefficients for each indicator for each student

| *Student number* | *Classroom performance* | *Assignment grades* | *Curriculum design* | *Mid-term test* | *Final examination* |
|---|---|---|---|---|---|
| 1 | 0.5586 | 0.6111 | 0.5052 | 0.3346 | 0.4414 |
| 2 | 0.5495 | 0.6191 | 0.5052 | 0.6162 | 0.5957 |
| 3 | 0.5405 | 0.6191 | 0.4947 | 0.6087 | 0.5212 |
| 4 | 0.5225 | 0.5714 | 0.5052 | 0.6237 | 0.5106 |
| 5 | 0.5135 | 0.6191 | 0.4947 | 0.5787 | 0.6968 |
| 6 | 0.4594 | 0.6191 | 0.5052 | 0.5937 | 0.5319 |
| … | … | … | … | … | … |
| 94 | 0.5676 | 0.6191 | 0.4947 | 0.4812 | 0.4414 |

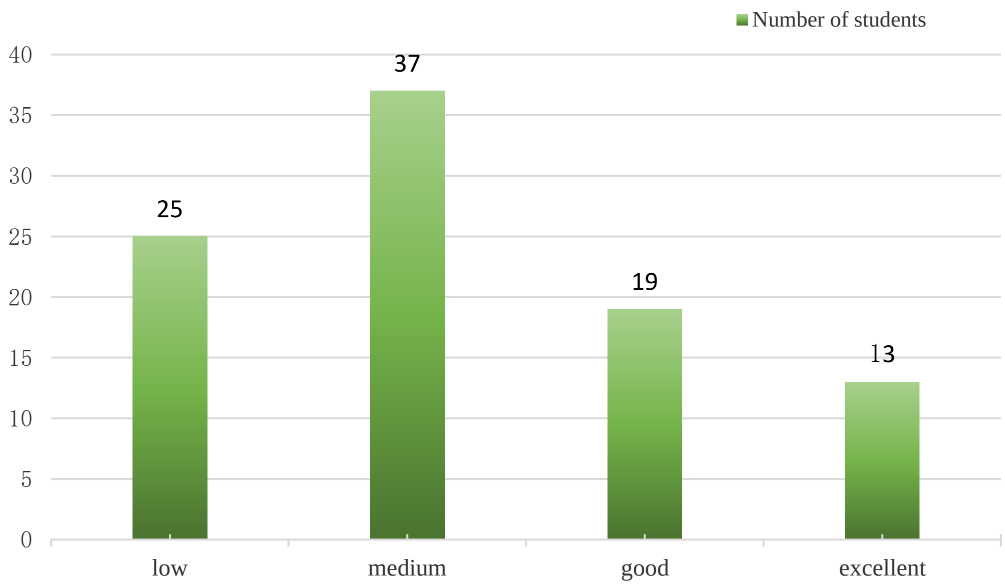


**Fig 1.** Clustering results of student performance

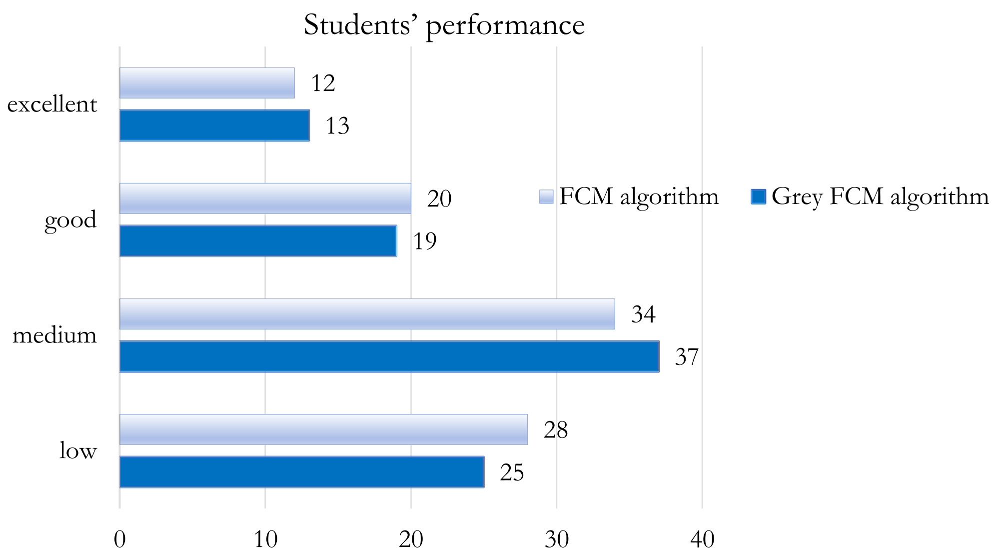


**Fig 2.** Comparison of clustering results of two algorithms

**Table 4.** Results of clustered grades for student course ratings

| Student Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | ... | 94 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Clustering results | 1 | 3 | 2 | 2 | 4 | 2 | 3 | 3 | 4 | 2 | ... | 1 |

level, there are 28 students, accounting for about 29.7% of the total number of students, and the proportion of the number of students is still large. From the analysis of the normalized data, the classroom performance, homework grades, and course design grades of these students are higher, while the examination grades are relatively low, which leads to unsatisfactory results of the student's course evaluation. These students should consolidate the learning of basic knowledge and consolidate the foundation through the practice of homework after class. Teachers should pay more attention to such students, play the guiding work of teachers, create situations for students, stimulate students' interest in learning, pay attention to the students' ordinary homework, and provide counselling. The number of students whose course grades were at the average level of the grade was 34, accounting for about 36.1% of the total number of students, which is the largest number of students among all the clusters accounted for. For this type of students, students must consolidate their review after the class is over, practice diligently, pay attention to the correct rate, get to the bottom of the problem, and ask the teacher for more advice. Teachers should adjust the pace of the course to the cognitive acceptance of this type of students, they are the main body of the whole class, determining the level of a class. The number of students with good grades in the course is 20, accounting for about 20.6%, the proportion of good grades in the course is not a lot of such students are more skilled in grasping the basics, but lack attention to some of the details, in the process of learning, attention should be paid to easy to error, to pay more careful attention to some of the traps in the thinking, you can sort out the wrong questions to improve the degree of their attentiveness. Teachers should give some guidance, and heuristic teaching, but also play the student's subjectivity so that such students develop the habit of active thinking. The number of students whose final course results were graded excellent was 12. These students have a very good grasp of the details and knowledge points, the teacher should carry out a certain amount of upgrading and expansion of thinking, in the classroom, through the way of questioning to stimulate their thinking, and enlightenment to let them form developmental thinking. This kind of student should usually read more to expand the information, not be confined to the knowledge of the textbook, more associations, more thinking, more and more teachers to exchange ideas, to cultivate students' creative ability.

## 4. Discussion

The comprehensive evaluation system of student performance created in this paper not only provides a new evaluation method for student evaluation but also provides a new perspective for teachers to judge students' course learning with a more objective and quantitative perspective, which is of some significance. The algorithm created in this paper can be applied not only to the comprehensive evaluation of student performance but also to other dimensions of the comprehensive evaluation of students, is still applicable. The algorithm created in this paper is a grey FCM-weighted comprehensive evaluation method based on the G1 method and entropy weighting method. Although the G1 method reflects the experience of experts, it has a certain degree of subjectivity, and this paper overcomes the shortcomings by introducing the entropy weighting method. The FCM clustering evaluation method can deal with the ambiguity of the sample data relative to the traditional hard clustering and can be used to deal with too much evaluation sample data in a fuzzy situation, which can be classified by the clustering algorithm according to the distance of samples to the center of clustering. The clustering algorithm can divide the samples into categories according to the distance to the cluster center, which is of significant significance when used to deal with the evaluation of too much sample data, and the introduction of grey correlation and weighted Euclidean distance, which consider both the influence of indicator weights and the non-linear relationship between the data, making the results more objective.

The evaluation designed in this paper mainly used the quantitative method, so that teachers can understand students' learning performance quantitatively. However, the evaluation only focuses on students' grades, to more accurately reflect students' learning and development, it’s better to combine qualitative evaluation with quantitative evaluation, and add students' developmental evaluation and classroom performance, such as the number of answers, the degree of thinking expansion, and classroom hands-on practice, in addition to their academic performance, to reflect students' course performance more comprehensively. Diverse evaluations can stimulate students' learning motivation and motivation. What’s more, student performance evaluation reflects the learning situation of students in a learning stage. After evaluation, it is important to give feedback, including students' feedback on the teacher's class, opinions on the course, teachers' evaluation, and self-reflection to have a profound understanding of the problems and shortcomings in their learning. Teachers should actively listen to the suggestions of leaders and students, only in this way can help improve the quality of teaching.

## 5. Conclusion

Comprehensive evaluation of students is crucial to the development of students and teachers, and it is important to construct a set of scientific, reasonable, and objective comprehensive evaluation systems for students to improve the quality of talent cultivation. The GEG-FCM comprehensive evaluation algorithm proposed in this paper considers both subjective and objective weights. At the same time, the grey FCM weighted clustering algorithm can divide the large-scale student samples into several clusters, so that teachers can carry out targeted teaching for students at different levels, and promote the personalized development of students. Teachers can apply the algorithm to a variety of evaluation systems for comprehensive evaluation of students, such as comprehensive evaluation of performance, comprehensive evaluation of learning ability, and so on, according to the actual evaluation needs. How to rationalize the evaluation indicators, adopting indicators more in line with the principle of developmental evaluation, and optimizing the model parameters and algorithms are the focus of the subsequent research in this paper.

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## Originality statement

The authors declare that the work reported in the current study is original, and no content (concept, text, tables, illustrations, data, etc.) supposed to be produced/generated/estimated/written/collected by the authors in the current study is partially or completely generated through Artificial Intelligence (AI) or any AI-based software.

## References

Deng, J. (1989). Introduction to Grey system theory. *The Journal of Grey System*, *1*(1), 1-24. https://dl.acm.org/doi/abs/10.5555/90757.90758

Gu, X., Wang, X., & Liang, S. (2023). Employment quality evaluation model based on hybrid intelligent algorithm. *Computers, Materials & Continua*, *74*(1), 131-139. https://doi.org/10.32604/cmc.2023.028756

Kou, X., Xie, X., Zou, Y., Kang, Q., & Liu, Q. (2022). Research on comprehensive evaluation model of a truck dispatching system in open-pit mine. *Sustainability*, *14*(15): 9062.https://doi.org/10.3390/su14159062

Liu, J., Pan, Y., & Yang, J. (2012, December). *Research on comprehensive evaluation of the students' vocational ability based on AHP-FUZZY*. In 2012 International Conference on Control Engineering and Communication Technology (pp. 791-793). IEEE. https://doi.org//10.1109/ICCECT.2012.242

Luo, Y., Jiang, F., Liu, K., Zhang, X., & He, G. (2022, July). *Health status assessment of distributed low-carbon energy station based on Grey-TOPSIS improved by prospect theory*. In 2022 IEEE/IAS Industrial and Commercial Power System Asia (I&CPS Asia) (pp. 521-526). IEEE. https://doi.org/10.1109/ICPSAsia55496.2022.9949750

Peizhuang, W. (1983). Pattern recognition with fuzzy objective function algorithms (James C. Bezdek). *Siam Review*, *25*(3), 442-442. https://doi.org/10.1137/1025116

Ren, X., Jiang, Q., Jiang, J., He, Z., Ouyang, B., & Peng, B. (2022). Evaluation of cabin energy consumption based on combination weighting and grey fuzzy comprehensive model. *EURASIP Journal on Advances in Signal Processing*, *2022*(1), 36. https://doi.org/10.1186/s13634-022-00858-3

Rosadi, R., Sudrajat, R., Kharismawan, B., & Hambali, Y. A. (2017). *Student academic performance analysis using fuzzy c-means clustering*. In IOP Conference Series: Materials Science and Engineering (Vol. 166, No. 1, pp. 012036). IOP Publishing. https://doi.org//10.1088/1757-899X/166/1/012036

Shannon, C. E. (1948). A mathematical theory of communication. *The Bell System Technical Journal*, *27*(3), 379-423. https://doi.org/10.1002/j.1538-7305.1948.tb01338.x

von Neumann, J., & Morgenstern, O. (1947). *Theory of games and economic behavior* (2nd rev. ed.). Princeton University Press. https://psycnet.apa.org/record/1947-03159-000

Wang, Z. (2022). Higher education management and student achievement assessment method based on clustering algorithm. *Computational Intelligence and Neuroscience*, *2022*, 1-10. https://doi.org/10.1155/2022/4703975

Wu, Y., Xia, X., & Le, J. (2017). *Evaluation of students' achievements based on factor analysis and cluster analysis*. In 2017 8th IEEE International Conference on Software Engineering and Service Science (ICSESS) (pp. 820-823). IEEE. https://doi.org//10.1109/ICSESS.2017.8343037

Ye, F., Sun, J., Wang, Y., Nedjah, N., & Bu, W. (2023). A novel method for the performance evaluation of institutionalized collaborative innovation using an improved G1-CRITIC comprehensive evaluation model. J*ournal of Innovation & Knowledge*, *8*(1), 100289. https://doi.org/10.1016/j.jik.2022.100289

Zhao, J. (2023). A method of power supply health state estimation based on grey clustering and fuzzy comprehensive evaluation. *IEEE Access*, *11*, 12226-12236. https://doi.org/10.1109/ACCESS.2023.3240692

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