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Grey Multiple-Criteria Decision-Making

Michael Gr. Voskoglou[1,*]

[1] *Professor Emeritus of Mathematical Sciences, School of Engineering, University of Peloponnese, Patras, Greece – 26334*

*Corresponding author: voskoglou@teiwest.gr

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Abstract: Decision Making (DM) is one of the most important components of human cognition. In particular, the Multiple-Criteria DM (MCDM), is a composite form of DM evaluating options with conflicting goals and choosing the best solution among the existing ones. Following the fuzzy DM criterion of Bellman and Zadeh in 1970, several other methods have been developed by other researchers for DM in fuzzy environments. Here we present a parametric, MCDM method utilizing grey numbers as tools. This method improves an earlier approach of Maji and colleagues in 2002, who used the tabular representation of a soft set as a tool for parametric MCDM in a fuzzy environment. The method is also extended to cover cases of weighted DM and suitable examples are presented illustrating our results.

Keywords: Grey Number; Soft Set; Tabular Representation; Decision-Making; Multiple-Criteria Decision Analysis

1. Introduction

Decision Making (DM), which is one of the most important components of human cognition, is the process of choosing a solution between two or more alternatives, on the purpose of achieving the optimal result for a given problem. Obviously DM has sense if, and only if, there exist more than one feasible solutions, together with one or more suitable criteria helping the decision maker to choose the best among these solutions. We recall that a solution is characterized as feasible, if it satisfies all the restrictions imposed onto the real system by the statement of the problem as well as all the natural restrictions imposed onto the problem by the real system; e.g. if x denotes the quantity of the stock of a product, it must be $x \geq 0$. The choice of the suitable criterion (or criteria), especially when the results of DM are affected by random events, depends upon the desired goals of the decision maker; e.g. optimistic or conservative criteria, etc.

The rapid technological progress, the impressive development of the transportation means, the globalization of human society, the continuous changes appearing to the local and international economies, and other related reasons, led during the last 60-70 years to a continuously increasing complexity of the problems of our everyday life. As a result the DM process became in many cases a very difficult task, which is impossible to be based on the decision maker's experience, intuition and skills only, as it usually happened in the past. Thus, from the beginning of the 1950's a progressive development started of a systematic methodology for the DM process, termed as Statistical Decision Theory, which is based on principles of Probability Theory, Statistics, Economics, Psychology and other related scientific topics (Berger, 1980).

The DM process involves the following steps:

- d1: Analysis of the decision problem, i.e. understanding, simplifying and reformulating the problem in a form permitting the application of the standard DM techniques on it.
- d2: Collection and interpretation of all the necessary information related to the problem.
- d3: Determination of all the feasible solutions.
- d4: Choice of the best solution in terms of the suitable, according to the decision maker's goals, criterion (or criteria).

One could add one more step to the DM process, the verification of the chosen decision according to the results obtained by applying it in practice. However, this step is extended to areas which, due to their depth and importance, have become autonomous. Therefore, it is usually examined separately from the other steps of the DM process (e.g. see Voskoglou, 2014).

Note that the first three steps of the DM process are continuous, in the sense that the completion of each one of them usually needs some time, during which the decision maker's reasoning is characterized by transitions between hierarchically neighbouring steps. In other words, the DM process, the flow-diagram of which is represented in Figure 1, cannot be characterized as a linear process.

In particular, the Multiple-Criteria Decision Making (MCDM), is a composite form of DM evaluating options with conflicting goals and choosing the best solution (e.g. see Taherdoost & Madanchian, 2023).

DM problems appear frequently in everyday life characterized by vagueness. In such cases the classical Statistical Decision theory does not offer effective tools for studying the DM process. Fuzzy Logic (FL), on the contrary, due to its nature of including multiple values, offers a rich field of resources for this purpose. Bellman and Zadeh (1970) were the first who applied principles of FL to DM.

Following the fuzzy DM criterion of Bellman and Zadeh, several other methods were proposed by other researchers for DM in fuzzy environments; e.g. Alcantud (2018), Alazemi *et al.* (2021), Zhu and Ren (2022), Khan *et al.* (2022), Chiclana *et al.* (1998), Ekel (2001, 2002), Ekel *et al.* (2016), etc. Here we will develop a parametric, MCDM method using soft sets (SSs), and grey numbers (GNs) as tools (Voskoglou, 2023a). This method improves an earlier method of Maji *et al.* (2002), which uses the tabular representation of a SS as a tool for MCDM in a fuzzy environment.

The present study is formulated as follows: Section 2 contains the necessary mathematical background about GNs and SSs, as well as the description, through a suitable example, of the parametric MCDM method of Maji *et al.* (2002). Section 3, after pointing out the weaknesses of the previous method, presents the improved method using GNs as tools, which results in better decisions than the method of Maji *et al.* when at least one of the parameters involved has a fuzzy texture. Section 4 discusses the weighted MCDM and the study closes with Section 5 including the final conclusions and some hints for future research.

2. Mathematical Background

2.1 Grey Numbers

The grey system (GS) theory was introduced by Deng (1982) for handling approximate data (Liu & Lin, 2010). The main tool for handling the approximate data of a GS is the use of GNs. Modern readers often find GS theory as a nonparametric alternative to fuzzy set theory (Zadeh, 1965) because of its flexible propositions and arithmetic.

$$\boxed{\text{d1}} \leftrightarrow \boxed{\text{d2}} \leftrightarrow \boxed{\text{d3}} \rightarrow \boxed{\text{d4}}$$

Fig 1. The flow-diagram of the decision-making process

DEFINITION 1: A GN L, denoted with $\otimes L$ is understood to be a real number with known range given by a closed real interval of the form $[a, b]$, but with unknown exact value. The GN $\otimes L$, however, may differ from the interval $[a, b]$ with respect to the presence of a whitenization function $f: [a, b] \to [0,1]$, such that the closer is $f(t)$ to 1, the better $t \in [a, b]$ approximates the unknown value of $\otimes L$.

When no such function exists, it is logical to consider as the crisp representative of L the real number

$$\mathrm{K}(\otimes L) = \frac{a+b}{2} \tag{1}$$

The real number $\mathrm{K}(\otimes L)$ is usually referred to as the kernel of $\otimes L$ and the process of calculating its kernel is usually referred to as the whitenization of $\otimes L$.

The known arithmetic of the real intervals (Moore et al, 1995) is used to perform the basic arithmetic operations between GNs. Let $\otimes L_1 \in [a_1, b_1]$ and $\otimes L_2 \in [a_2, b_2]$ be given GNs and let r be a positive number. In this paper we will make use only of the addition and of the scalar product of GNs, which are defined respectively by the relations

$$\otimes L_1 + \otimes L_2 \in [a_1 + b_1, a_2 + b_2] \tag{2}$$

and

$$r \otimes L_1 \in [ra_1, rb_1]. \tag{3}$$

2.2 Using Soft Sets for Parametric Decision Making

Molodtsov (1999) introduced the notion of SS for a parametric treatment of the real-world uncertainty in the following way:

DEFINITION 2: Let E be a set of parameters and let f be a map from E into the power set $P(U)$ of the universal set U. Then the SS (f, E) in U is defined as a parameterized family of subsets of U by

$$(f, E) = \{(e, f(e)) : e \in A\} \tag{4}$$

The term "soft" was introduced because the form of (f, E) depends on the parameters of E.

Maji *et al.* (2002) introduced the *tabular representation* of a SS for storing it easily in a computer's memory and they used it for parametric DM. The following example illustrates their DM methodology.

EXAMPLE 1: A company wants to employ a person among six candidates, say A_1, A_2, A_3, A_4, A_5 and A_6. The ideal qualifications for the new employee is to have satisfactory previous experience, to hold a university degree, to have a driving license and to be young. Assume that A_1, A_2, A_6 are the candidates with satisfactory previous experience, A_2, A_3, A_5, A_6 are the holders of a university degree, A_3, A_5 are the holders of a driving license and A_4 is the unique young candidate. Find the best decision for the company.

SOLUTION: Set $U = \{A_1, A_2, A_3, A_4, A_5, A_6\}$ and let $F = \{p_1, p_2, p_3, p_4\}$ be the set of the parameters p_1=well experienced, p_2=holder of a university degree, p_3=holder of a driving license and p_4=young. Consider the map $f: F \to P(U)$ defined by

$f(p1) = \{A_1, A_2, A_6\}, f(p2) = \{A_2, A_3, A_5, A_6\}, f(p3) = \{A_3, A_5\}, f(p4) = \{A_4\}$.

Then the SS defined with respect to F and f is equal to

$$(f, F) = \{(p1, \{A_1, A_2, A_6\}), (p2, \{A_2, A_3, A_5, A_6\}), (p3, \{A_3, A_5\}), (p4, \{A_4\})\}$$

The tabular representation of the SS (f, F), shown in Table 1, is formed by assigning to each candidate the binary element 1, if he/she satisfies the qualification addressed by the corresponding parameter, or the binary element 0 otherwise.

Then, the *choice value* V of each candidate is determined by adding the entries of the row of the tabular representation of (f, F) where it belongs. Thus, the candidates A_1 and A_4 have choice

Table 1. Tabular representation of the SS (f, F) of Example 1

	p_1	p_2	p_3	p_4
A_1	1	0	0	0
A_2	1	1	0	0
A_3	0	1	1	0
A_4	0	0	0	1
A_5	0	1	1	0
A_6	1	1	0	0

value 1 and all the others 2. The company, therefore must employ one of the candidates A_2, A_3, A_5 or, A_6.

3. Grey Decision-Making

The DM method of Maji al. is not very helpful for the company in Example 1 to choose the new employee, since it excluded only two (A1 and A4) among the six candidates. This is due to the fact that in the tabular matrix of the corresponding SS the characterization of the candidates by the corresponding parameters was done by using the binary elements (truth values) 0, 1. In other words, although the method of Maji and colleagues starts from a fuzzy basis utilizing SSs as tools, then it uses bivalent logic for making the required decision. This could lead to inadequate decisions, if some of the parameters have a fuzzy texture, like it happens with the parameters p_1: well-experienced and p_4: young of Example 1. For tackling this problem, we have used GNs instead of the binary elements 0, 1 in the tabular representation of the corresponding SS (Voskoglou, 2023a). This methodology is illustrated here with the following example:

EXAMPLE 2: Revisit Example 1 and assume that the analysts of the company, after studying more carefully the available information for the six candidates, decided to use the GNs $G_1 \in [0.85, 1]$, $G_2 \in [0.75, 0.84]$, $G_3 \in [0.6, 0.74]$, $G_4 \in [0.5, 0.59]$ and $G_5 \in [0, 0.49]$ instead of the binary elements 0, 1 for characterizing the parameters p_1 and p_4, which have a fuzzy texture, as shown in Table 2. Which is the best decision for the company in this case?

SOLUTION: Adopting the notation used in the solution of Example 1, Table 2 gives the revised tabular representation of the SS (f, F). In this case the choice values are calculated through the whitenization of the GNs. Consequently, with the help of formulas (1) and (2) one finds that

$$V_1 = 0 + 0 + K(G_1 + G_3) = K([0.85 + 0.6, 1 + 0.74]) = \frac{1.45 + 1.74}{2} = 1.595$$

and similarly,

$$V_2 = 1 + 0 + K(G_1 + G_5) = 2.17,$$
$$V_3 = 1 + 1 + K(G_3 + G_3) = 3.34,$$
$$V_4 = 0 + 0 + K(G_4 + G_1) = 1.47,$$
$$V_5 = 1 + 1 + K(G_4 + G_3) = 3.215,$$
$$V_6 = 1 + 0 + K(G1 + G4) = 2.47.$$

Table 2. Characterizations of the parameters involved in Example 2

	p_1	p_2	p_3	p_4
A_1	$\otimes G_1$	0	0	$\otimes G_3$
A_2	$\otimes G_1$	1	0	$\otimes G_5$
A_3	$\otimes G_3$	1	1	$\otimes G_3$
A_4	$\otimes G_4$	0	0	$\otimes G_1$
A_5	$\otimes G_4$	1	1	$\otimes G_3$
A_6	$\otimes G_1$	1	0	$\otimes G_4$

Therefore, the best decision for the company is to choose the candidate A_3. This is obviously a better decision than that the one made in Example 1, because it leads to the choice of only one candidate (A_3) not creating dilemma for the company like it happened in the case presented in Example 1 (choice of one among the candidates A_2, A_3, A_5 and A_6).

4. Weighted Decision-Making

DM cases appear frequently in everyday life in which the decision maker's goals are not equally important. In such cases, weight coefficients, whose sum is equal to 1, are assigned to each parameter. This is illustrated here with the following example.

EXAMPLE 3: Revisit Examples 1 and 2 and assume that the weight coefficients 0.4, 0.3, 0.2 and 0.1 have been assigned to the parameters p_1, p_2, p_3 and p_4 respectively according to the importance of the goals of the company. Which is the best choice for the company under these conditions?

SOLUTION: In case of Example 1, after incorporating the weights, the new choice values of the candidates become 0.4 (A_1), 0.7 (A_2), 0.5 (A_3), 0.1 (A_4), 0.5 (A_5) and 0.7 (A_6). Therefore, the company must choose one of the candidates A_2 or A_6.

In case of Example 2, with the help of formulas (1), (2) and (3) one finds that the weighted choice value of the candidate A1 is equal to

$$V1 = K[0.4(G1) + 0.1(G3)] = 0.437.$$

Similarly

$$V2 = 0.3 + K[0.4(G1) + 0.1(G5)] = 0.6945,$$
$$V3 = 0.3 + 0.2 + K[0.4(G3) + 0.1(G3)] = 0.835,$$
$$V4 = K[0.4(G4) + 0.1(G1)] = 0.168,$$
$$V5 = 0.3 + 0.2 + K[0.4(G4) + 0.1(G3)] = 0.758,$$
$$V6 = 0.3 + K[0.4(G3) + 0.1(G4)] = 0.6225.$$

Therefore, the best decision for the company is to employ the candidate A_3.

In conclusion, if the analysts of the company are sure about the qualifications of the six candidates, following the weighted DM method of Maji *et al.* of the revised Example 1, must choose one of the candidates A_2 or A_6. Otherwise, following the weighted grey DM method of the revised Example 2 (i.e. using GNs in the decision matrix instead of the binary elements 0, 1) must choose the candidate A_3.

5. Discussion and conclusion

Following the introduction of the theory of fuzzy sets by Zadeh (1965), various extensions and related theories have been developed through the years for a more effective management of the existing in the real-world uncertainty; e.g. see Voskoglou (2019a). Each one of them is suitable for tackling one or more types of uncertainty, but none of them can tackle alone all the existing forms of it. All these theories together, however, form an adequate framework for managing the uncertainty in general.

Furthermore, suitable combinations of the previous theories seem to provide better results. In this work, for example, using a combination of SSs and GNs, we developed a hybrid model for parametric MCDM, which improves an earlier model of Maji *et al.* (2002) using only SSs as tools. Similar MCDM models were developed by the present author, in which the binary elements 0, 1 in the tabular matrix of the corresponding SS were replaced either by intuitionistic fuzzy pairs (Voskoglou, 2023b), or by neutrosophic triplets (Voskoglou, 2023c), depending on the form of the corresponding DM problem. Taking into account that analogous hybrid models were also developed for the assessment of several human or machine activities under fuzzy conditions (e.g. see Voskoglou, 2019a), one concludes that this is an interesting and much promising approach for further research.

Originality statement

The author(s) declares that the work reported in the current study is original, and no content (concept, text, tables, illustrations, data, etc.) supposed to be produced/generated/estimated/written/collected by the author(s) in the current study is partially or completely generated through Artificial Intelligence (AI) or any AI-based software.

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