International Journal of Grey Systems
ISSN 2767-6412 / eISSN 2767-3308
2023 Volume 3 Issue 2: 5–7
https://doi.org/10.52812/ijgs.77

Grey Assessment

Michael Gr. Voskoglou[1,*]
[1] *Professor Emeritus of Mathematical Sciences, School of Engineering, University of Peloponnese, Patras, Greece – 26334*
*Corresponding author: voskoglou@teiwest.gr

Received 23 July 2023; *Revised* 16 October 2023; *Accepted* 7 November 2023

Abstract: The assessment of human activities is an important task, because it helps to correct mistakes and to improve the overall performance of a group. Frequently, however, the individual assessment of the members of a group is performed not with numerical scores, but with qualitative grades (linguistic expressions). This happens either because the existing data are not exact, or for reasons of elasticity (e.g. teacher for students). In such cases the mean performance of the group cannot be assessed by applying the traditional method of calculating the mean value of the individual scores of its members. In this short communication, a method developed recently by the author is presented for evaluating the mean performance of a group in such cases, using grey numbers as tools. The method is illustrated by an example concerning the assessment of the mean performance of a class of students.

Keywords: Qualitative assessment; mean performance of a group; linguistics; grey number; grey assessment

1. Introduction

Frequently in everyday life the assessment of a group's performance takes place by using qualitative grades (linguistic expressions) instead of numerical scores. This is due to various reasons, the main of which is either the existence of non-exact data, or the will for more elasticity (e.g. from teacher to students).

In this short communication, we present a method, developed in the author's earlier works (e.g. Voskoglou, 2019: section 6.2), for evaluating the mean performance of a group, when qualitative grades are used for assessing the individual performance of its members. This method uses grey numbers (GNs) as tools and is illustrated by an example concerning the assessment of the mean performance of a class of students.

The theory of grey systems (GSs) (Deng, 1982) is an alternative to the theory of fuzzy sets (Zadeh, 1965) for handling approximate data. A GS is defined to be any investigated system with poor information concerning its structure message, operation mechanism, behaviour document, etc. The theory of GSs was developed mainly in China and has found many important applications to everyday life, science and engineering, including medicine diagnostics, psychology, sociology, control systems, economics, agriculture, opinion polls, etc., where the data cannot be easily determined and estimates of them are used in practice. For general facts on GSs we refer to Liu and Lin (2010).

The main tool for handling the approximate data of a GS is the use of GNs. A GN T, denoted with $\otimes T$, is understood to be a real number with known range given by a closed real interval of the form $[a, b]$, but with unknown exact value. The GN $\otimes T$, however, may differ from the interval $[a, b]$ with respect to the presence of a whitenization function $f: [a, b] \to [0,1]$, such that the closer is $f(t)$ to 1, the better $t \in [a, b]$ approximates the unknown value of $\otimes T$. When no such function exists, it is logical to consider as the crisp representative (kernel) of $\otimes T$ the real number

$$\mathrm{V}(\otimes T) = \frac{a+b}{2} \tag{1}$$

The known arithmetic of the real intervals (Moore *et al.*, 1995) is used to perform the basic arithmetic operations between GNs. Let $\otimes T_1 \in [a_1, b_1]$ and $\otimes T_2 \in [a_2, b_2]$ be given GNs and let r be a positive number. In this paper we will make use only of the addition and of the scalar product of GNs, which are defined respectively by the relations,

$$\otimes T_1 + \otimes T_2 \in [x_1 + y_1, x_2 + y_2] \tag{2}$$

and

$$r \otimes T_1 \in [rx_1 + ry_1]. \tag{3}$$

2. Methodology

A commonly used scale of qualitative grades in assessment processes is: A = *excellent*, B = *very good*, C = *good*, D = *fair* and F= *fail*. In certain cases, the grade E is also inserted between D and F, or intermediate grades like A_-, B_+, B_-, etc. are used, but this does not affect the generality of our method.

We assign the numerical scale $1 - 100$ to the previous qualitative grades as follows: $A \to [85, 100]$, $B \to [75, 84]$, $C \to [60, 74]$, $D \to [50, 59]$, $F \to [0, 49]$. This assignment, although it is compatible with the common sense, is not unique. For a stricter assessment, for example, one could consider instead the assignment $A \to [90, 100]$, $B \to [80, 89]$, $C \to [70, 79]$, $D \to [60, 69]$, $F \to [0, 59]$, etc. Neither this fact, however, affects the generality of our method.

We now introduce the following GNs, denoted for simplicity with the same letters: $\otimes A \in [85, 100]$, $\otimes B \in [75, 84]$, $\otimes C \in [60, 74]$, $\otimes D \in [50, 59]$, $\otimes F \in [0, 49]$.

Let us consider a group G of n objects under assessment. Assume that the performance of n_A of these objects was evaluated with A, of n_B with B, of n_C with C, of n_D with D and of n_F objects with F, so that $n_A + n_B + n_C + n_D + n_F = n$. With the help of equations (2) and (3) we define the mean value of the corresponding GNs to be the GN

$$\otimes M = \frac{1}{n}(n_A \otimes A + n_B \otimes B + n_C \otimes C + n_D \otimes D + n_F \otimes F) \tag{4}$$

Then the mean performance of the group G can be estimated, with the help of equation (1), by the real value $V(\otimes M)$.

3. Numerical example

Our previous assessment method is illustrated here with the following example:

EXAMPLE: The teacher of a class of twenty students assessed the performance of his students as follows: Students $s_1 - s_3$ with A, $s_4 - s_7$ with B, $s_8 - s_{10}$ with C, $s_{11} - s_{16}$ with D, and the remaining four students with F. Evaluate the mean performance of the class.

SOLUTION: With the help of equation (4) one finds that the mean value of the grades obtained by the students of the class is equal to

$$\otimes M = \frac{1}{20}\{3[85, 100] + 4[75, 84] + 3[60, 74] + 6[50,59] + 4[0,49]\}.$$

Therefore, with the help of equations (2) and (3), it turns out that

$$\otimes M = \frac{1}{20}[1035, 1408] = [51.75, 70.4].$$

Thus, by equation (1), one finds that $V(\otimes M) = \frac{51.75+70.4}{2} = 61.075$, which shows that the student class demonstrated a good (C) mean performance.

The closed network diagram (Figure 1) explains our methodology very well showing that the algorithm starts from linguistic input (linguistic scale) and ends with a linguistic output (in our example with C = good performance).

Linguistics → Grey Numbers → Mean of corresponding Grey Numbers → Kernel → Linguistics

Fig 1. A graphical representation of our assessment methodology

4. Discussion and conclusion

In the current study, we presented a method using GNs as tools, for estimating the mean performance of a group of objects with respect to a certain activity, when their individual performance is assessed with qualitative grades. In such cases the group's mean performance cannot be evaluated by applying the traditional method of calculating the mean value of the individual scores. An alternative method for estimating a group's mean performance in such cases is by using triangular fuzzy numbers (TFNs) as tools, instead of GNs. We have shown, however, that these two methods are equivalent to each other (Voskoglou, 2019: paragraphs 5.2 and 6.2), the method with the GNs being simpler.

A similar method using GNs can be also applied for decision making (DM) (Voskoglou, 2023). This method improves an earlier DM method of Maji *et al.* (2002) using soft sets as tools.

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