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A Conformable Fractional Discrete Grey Model CFDGM (1,1) and its Application

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Abstract: An accurate forecast of the area of drought disaster is vitally important for the government to take appropriate measures to prevent disaster. In the current study, a new conformable fractional discrete grey model is applied to study the trend of the area affected by drought disasters. Firstly, the new model, abbreviated as CFDGM(1,1), is proposed with the definitions of the conformable fractional operator and the classical GM(1,1) model. Then the recursive expression of the time response function is obtained by the grey basic form, and the linear system parameters are confirmed by the linear least squares method. Further, the Salp Swarm Algorithm is chosen to determine the optimal conformable fractional order. Finally, the area of drought disaster is studied by the new model and others, where the results show the new model has a good performance among these models.

Keywords: Drought disaster; grey forecast model; conformable fractional operator; salp swarm algorithm

1. Introduction

Drought as a natural disaster happens due to climate variability under different climatic conditions on all continents and has a disastrous impact on the ecosystems, agricultural production, and economic and social conditions. Drought disaster is a serious problem in China, and the causes of drought disasters in various parts of China can be roughly summarised into three aspects. The first aspect is the precipitation, where the precipitation is lower than average in most places. The second aspect is the water resources because of the imbalance of water resources in different regions in China. The third aspect is the socio-economic factors such as the increasing water consumption in industrial and agricultural production in China in recent years. Due to these factors, the drought disasters happened, and the area of the drought disaster was also influenced.

To study the area of drought disaster, the current study uses the grey forecasting models. The grey theory was proposed by professor Deng (1982), and the grey system refers to an incomplete information system with partially clear and partially unclear information. The grey predicting theory is an important part of the grey system, and it does not require a lot of data for modeling to achieve accurate results. The first-order univariate grey forecasting model GM(1,1) is the core of grey forecasting models, where the time response function is derived by its whitening differential equation and the system parameters are derived by the grey basic form. To improve the precision of the GM(1,1) model, Javed and Cudjoe (2022) considered DGM(1,1,α) model and applied to forecast the emissions in four industries sectors of China and India. Gou *et al.* (2022) proposed a new high-performance grey prediction model FDGM(1,1, $\sqrt{k}$, r) for wastewater discharge prediction. Gao *et al.* (2022) studied a novel grey Gompertz model FAGGM(1,1) for carbon emission forecasting in the American industrial sector. Ma *et al.* (2020) first proposed a new conformable fractional grey model CFGM(1,1), in which the conformable fractional operator is used for the pretreatment of raw data. The conformable fractional operator is much easier for theoretical analysis and applications than the classical fractional operator. With a series of numerical examples, the results demonstrate that the new model is more efficient in non-smooth time series prediction and longer-term forecasting than the classical fractional grey model FGM(1,1) proposed by Wu *et al.* (2013). Subsequently, Wu *et al.* (2020) studied a conformable fractional non-homogeneous grey model and used it to forecast the carbon dioxide emissions of BRICS countries. The results obtained by the newly constructed model are compared with the models NGM(1,1,k,c), NGMO(1,1,k,c), FGM(1,1), FANGM(1,1,k,c) and showed that the model CFNGM(1,1,k,c) out-performed others in terms of forecast accuracy.

Xie *et al.* (2020a) proposed a continuous conformable fractional grey model CCFGM(1,1) with the definition of conformable fractional derivative and then applied it to the domestic energy consumption of China and the domestic coal consumption of China. Xie *et al.* (2020b) studied the annual electricity consumption of China by a CFGOM(1,1) with the opposite direction. Zheng *et al.* (2021) proposed a conformable fractional non-homogeneous grey Bernoulli model to study natural gas production and consumption. Liu *et al.* (2021) considered Jiangsu's electricity consumption with two types of conformable fractional grey interval models. Due to the conformable fractional operator being simple and easy to implement, there have been considerable pieces of literature on conformable fractional grey models during the last two years, see Wu *et al.* (2022a; 2022b; 2019), Xie *et al.* (2021; 2020c), and Xu *et al.* (2020). These works enriched and improved the conformable fraction grey models in theory and applications.

In the CFGM(1,1) model (Ma *et al.*, 2020), the time response function is obtained by solving the whitening differential equation, and the linear system parameters are derived by the grey basic form. However, the whitening differential equation and the grey basic form of the CFGM(1,1) model are inconsistent because the background values are deduced with the help of the trapezoidal approximation formula. This treatment may cause large errors in some applications. Therefore, inspired by the above research work, this study introduces the conformable fractional accumulation and difference into the classical univariate discrete grey model to construct a new grey model called CFDGM(1,1) and then applies it to the area of drought disaster.

The rest of this paper is organised as follows. The next section systematically studies the CFDGM(1,1) model. We first give the definition of the conformable fractional accumulation and difference and then give the definition of the new model. With the theory of the ordinary differential equations, the least squares estimation method, and the salp swarm algorithm, the expressions of the time response function and system parameters are determined. Section 3 studies the area of drought disaster in China. Conclusions are drawn in the last section.

2. The conformable fractional discrete grey model

The definition of the conformable fractional operator, and the properties of the proposed models are discussed in this section.

2.1 The conformable fractional operator

This subsection introduces the definition of the conformable fractional operator, including the conformable fractional accumulation and conformable fractional difference.

DEFINITION 1 (Wu *et al.*, 2020). The expression of the conformable fractional accumulation is given by

$$\nabla^{\alpha} f(k)=\sum_{i=1}^{k}\binom{k-i+\lceil\alpha\rceil-1}{k-i}\frac{f(i)}{i^{\lceil\alpha\rceil-\alpha}},\alpha>0, \tag{1}$$

where the combine number $\binom{k-i+\lceil\alpha\rceil-1}{k-i}=\frac{(k-i+\lceil\alpha\rceil-1)!}{(k-1)!(\lceil\alpha\rceil-1)!}$, and $\lceil\alpha\rceil$ is the smallest integer greater than or equal to α.

DEFINITION 2 (Wu *et al.*, 2020). The expression of the conformable fractional difference is given by

$$\Delta^{\alpha} f(k)=k^{\lceil\alpha\rceil-\alpha}\binom{\lceil\alpha\rceil}{k-i}f(i),\alpha>0, \tag{2}$$

where $\binom{\lceil\alpha\rceil}{k-i}=\frac{\lceil\alpha\rceil!}{(\lceil\alpha\rceil-k+1)!(k-i)!}$.

DEFINITION 3 (Ma *et al.*, 2020). Let sequence be $X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(n)\right)$, the α^{th} conformable fractional accumulation (α-CFA) sequence is $X^{(\alpha)}=\left(x^{(\alpha)}(1),x^{(\alpha)}(2),\ldots,x^{(\alpha)}(n)\right)$, and the α^{th} conformable fractional difference (α-CFD) sequence is $X^{(-\alpha)}=\left(x^{(-\alpha)}(1),x^{(-\alpha)}(2),\ldots,x^{(-\alpha)}(n)\right)$. The expression of them are outlined as follows.

$$x^{(\alpha)}(k)=\sum_{i=1}^{k}\begin{bmatrix}\lceil\alpha\rceil\\ k-i\end{bmatrix}\frac{1}{i^{\lceil\alpha\rceil-\alpha}}x^{(0)}(i),\alpha>0\ \left(k=1,2,\ldots,n\right), \tag{3}$$

$$x^{(-\alpha)}(k)=k^{\lceil\alpha\rceil-\alpha}\sum_{i=1}^{k}\begin{bmatrix}-\lceil\alpha\rceil\\ k-i\end{bmatrix}x^{(0)}(i),\alpha>0\ \left(k=1,2,\ldots,n\right). \tag{4}$$

2.2 The CFDGM (1, 1) model

DEFINITION 4. With the classical grey prediction model GM (1,1) and the conformable fractional accumulation, the whitening differential equation of the new model is

$$\frac{dx^{(\alpha)}(t)}{dt}+b_1x^{(\alpha)}(t)=b_2, \tag{5}$$

where b_1 is the development coefficient, b_2 is the grey action quantity, and α is the conformable fractional order accumulation.

Integrating both sides of the whitening differential Eq. (5) on the interval [k-1, k], it can calculate that

$$\int_{k-1}^{k}dx^{(\alpha)}(t)+b_1\int_{k-1}^{k}x^{(\alpha)}(t)dt=b_2\int_{k-1}^{k}dt. \tag{6}$$

Solving Eq. (6) with the trapezoidal approximation formula, it can be simplified that

$$x^{(\alpha)}(k)-x^{(\alpha)}(k-1)+b_1\frac{x^{(\alpha)}(k)+x^{(\alpha)}(k-1)}{2}=b_2, \tag{7}$$

which also arrivals

$$(2+b_1)x^{(\alpha)}(k)-(2-b_1)x^{(\alpha)}(k-1)=2b_2. \tag{8}$$

Thus, it is obvious that

$$x^{(\alpha)}(k)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(k-1)=\frac{2b_2}{2+b_1}, \tag{9}$$

which is the grey basic form of the whitening differential equation. And then, we will deduce the expression of the model with Eq. (9), which is called conformable fractional discrete grey model CFDGM(1,1).

THEOREM 1. The recursive formula of the time response function of the CFDGM(1,1) model is

$$x^{(\alpha)}(k)=\left(\frac{2-b_1}{2+b_1}\right)^{k-1}x^{(\alpha)}(1)+\frac{b_2}{b_1}\left[1-\left(\frac{2-b_1}{2+b_1}\right)^{k-1}\right],\ k=2,3,\ldots,. \tag{10}$$

and the restored values are

$$x^{(-\alpha)}(k)=k^{\lceil\alpha\rceil-\alpha}\sum_{i=1}^{k}\begin{bmatrix}-\lceil\alpha\rceil\\ k-i\end{bmatrix}x^{(0)}(i),\alpha>0,\ k=1,2,\ldots,n. \tag{11}$$

PROOF 1. It follows from the grey basic form of the CFDGM(1,1) model that

$$\begin{gathered}
x^{(\alpha)}(k)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(k-1)=\frac{2b_2}{2+b_1},\\
x^{(\alpha)}(k-1)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(k-2)=\frac{2b_2}{2+b_1},\\
x^{(\alpha)}(k-2)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(k-3)=\frac{2b_2}{2+b_1},\\
\vdots\\
x^{(\alpha)}(2)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(1)=\frac{2b_2}{2+b_1}.
\end{gathered}$$

Multiplying factors $\left(\frac{2-b_1}{2+b_1}\right)^{i}(i=0,1,\ldots,k-2)$ on both sides of this equations, we obtain the following set of equations

$$\begin{gathered}
\left(\frac{2-b_1}{2+b_1}\right)^{0}\left[x^{(\alpha)}(k)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(k-1)\right]=\left(\frac{2b_2}{2+b_1}\right)\left(\frac{2-b_1}{2+b_1}\right)^{0},\\
\left(\frac{2-b_1}{2+b_1}\right)^{1}\left[x^{(\alpha)}(k-1)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(k-2)\right]=\left(\frac{2b_2}{2+b_1}\right)\left(\frac{2-b_1}{2+b_1}\right)^{1},
\end{gathered}$$

$$\left(\frac{2-b_1}{2+b_1}\right)^2\left[x^{(\alpha)}(k-2)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(k-3)\right]=\left(\frac{2b_2}{2+b_1}\right)\left(\frac{2-b_1}{2+b_1}\right)^2$$

$$\vdots$$

$$\left(\frac{2-b_1}{2+b_1}\right)^{k-2}\left[x^{(\alpha)}(2)-\frac{2-b_1}{2+b_1}x^{(\alpha)}(1)\right]=\left(\frac{2b_2}{2+b_1}\right)\left(\frac{2-b_1}{2+b_1}\right)^{k-2},$$

then we add these equations together to produce the following results

$$x^{(\alpha)}(k)=\left(\frac{2-b_1}{2+b_1}\right)^{k-1}x^{(\alpha)}(1)+\frac{b_2}{b_1}\left[1-\left(\frac{2-b_1}{2+b_1}\right)^{k-1}\right],\ k=2,3,\ldots,$$

and then we complete this proof by the conformable fractional operator.

It can be seen that the system parameters b_1 and b_2 in Eqs. (10) and (11) are unknown and needs to be determined. Thus the following theorem gives the expression of system linear parameters b_1 and b_2.

THEOREM 2. The system linear parameters b_1 and b_2 of the CFDGM (1, 1) model can be expressed as follows.

$$(b_1,b_2)^T=\left(B^TB\right)^{-1}\left(B^TY\right), \tag{12}$$

where the matrices B and Y are

$$B=\begin{pmatrix}\frac{x^{(\alpha)}(1)+x^{(\alpha)}(2)}{2} & -1\\ \frac{x^{(\alpha)}(2)+x^{(\alpha)}(3)}{2} & -1\\ \vdots & \vdots\\ \frac{x^{(\alpha)}(r-1)+x^{(\alpha)}(r)}{2} & -1\end{pmatrix},\ Y=\begin{pmatrix}x^{(\alpha)}(1)-x^{(\alpha)}(2)\\ x^{(\alpha)}(2)-x^{(\alpha)}(3)\\ \vdots\\ x^{(\alpha)}(r-1)-x^{(\alpha)}(r)\end{pmatrix}.$$

PROOF 2. Integrating the whitening differential Eq. (5) and organising it, we obtain

$$\int_{k-1}^{k}dx^{(\alpha)}(t)+b_1\int_{k-1}^{k}x^{(\alpha)}(t)dt=b_2\int_{k-1}^{k}dt\,.$$

Applying the trapezoid approximation formula $\int_{k-1}^{k}x^{(\alpha)}(t)dt=\frac{x^{(\alpha)}(k)+x^{(\alpha)}(k-1)}{2}$, so it yields that

$$x^{(\alpha)}(k)-x^{(\alpha)}(k-1)+b_1\frac{x^{(\alpha)}(k)+x^{(\alpha)}(k-1)}{2}=b_2\,. \tag{12}$$

It is easily to achieve that

$$b_1\frac{x^{(\alpha)}(k)+x^{(\alpha)}(k-1)}{2}-b_2=x^{(\alpha)}(k)-x^{(\alpha)}(k-1).$$

Considering $k=2,3,\ldots,r$ in the above equation, and writing them in matrix form, it arrives that

$$\begin{pmatrix} \frac{x^{(\alpha)}(1)+x^{(\alpha)}(2)}{2} & -1 \\ \frac{x^{(\alpha)}(2)+x^{(\alpha)}(3)}{2} & -1 \\ \vdots & \vdots \\ \frac{x^{(\alpha)}(r-1)+x^{(\alpha)}(r)}{2} & -1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} x^{(\alpha)}(1)-x^{(\alpha)}(2) \\ x^{(\alpha)}(2)-x^{(\alpha)}(3) \\ \vdots \\ x^{(\alpha)}(r-1)-x^{(\alpha)}(r) \end{pmatrix},$$

and then the expression of system linear parameters of the new model is computed by

$$(b_1,b_2)^T = \left(B^T B\right)^{-1}\left(B^T Y\right).$$

2.3 The Salp Swarm Algorithm for optimal conformable fractional order

This subsection outlines the details of the search process for the optimal conformable fractional order α. At first, some measures are provided to show the feasibility and accuracy of grey forecasting models: the absolute percentage error (APE) and the mean absolute percentage error (MAPE). The mathematical formulas of them are produced below.

The absolute percentage error (APE)

$$APE(k) = \left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%, (k=2,3,\ldots). \tag{14}$$

The mean absolute percentage error (MAPE)

$$MAPE = \frac{1}{n}\sum_{k=1}^{n}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%, (k=2,3,\ldots). \tag{14}$$

It can be seen from the above analysis that the values of parameters b_1 and b_2 can be calculated by the least squares estimation method, and the remaining task is to find the value of the conformable fractional order. Thus an optimisation problem where α is a decision variable is constructed, where the corresponding objective function is given below.

$$\underset{\alpha}{MAPE} = \frac{1}{r}\sum_{k=1}^{r}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%,\ k=2,3,\ldots,$$

$$\text{s.}t.\begin{cases} (b_1,b_2)^T = \left(B^T B\right)^{-1}\left(B^T Y\right), \\ x^{(\alpha)}(k) = \left(\frac{2-b_1}{2+b_1}\right)^{k-1} x^{(\alpha)}(1) + \frac{b_2}{b_1}\left[1-\left(\frac{2-b_1}{2+b_1}\right)^{k-1}\right], k=2,3,\ldots \\ x^{(0)}(k) = k^{\lceil\alpha\rceil-\alpha}\sum_{i=1}^{k}\begin{bmatrix} -\lceil\alpha\rceil \\ k-i \end{bmatrix} x^{(\alpha)}(i), \alpha>0, k=1,2,\ldots,n. \end{cases}$$

It is known that it's an arduous task to derive the closed-form expression of the optimal conformable fractional order because this optimisation problem is highly nonlinear and complex. Therefore, the nature-inspired algorithm called Salp Swarm Algorithm (SSA) (Mirjalili *et al.*, 2017) is adapted to numerically find α. This algorithm is a new type of bionic swarm intelligence algorithm. The algorithm has some advantages, such as fewer adjustment parameters, easy-to-understand concepts, less difficulty in programming, and good directionality in optimisation. The pseudo-code of the Salp Swarm Algorithm is presented below.

Initialise the salp population $x_i(i=1,2,\ldots,n)$ considering upper bound *ub* and lower bound *lb* of α.
While (end condition is not satisfied)
Calculate the fitness of each search agent (salp)
F=the best search agent

Update c_1 by $c_1 = 2e^{-\left(\frac{4l}{L}\right)^2}$

for each salp (x_i)

if $(i == 1)$

Update the position of the leading salp by

$$x_j^1 = \begin{cases} F_j + c_1\left(\left(ub_j - lb_j\right)c_2 + lb_j\right) & c_3 \geq 0 \\ F_j - c_1\left(\left(ub_j - lb_j\right)c_2 + lb_j\right) & c_3 < 0 \end{cases}$$

else

Update the position of the follower salp by $x_j^i = \frac{1}{2}\left(x_j^i + x_j^{i-1}\right)$

end
end
Amend the salps based on the upper and lower bounds of variables
end
return F

3. Applications

This application section discusses the area of drought disaster in China by different grey forecasting models, including the DGM(1,1), FDGM(1,1) and CFDGM(1,1) models. The raw data of the area of drought disaster of China are all collected from the website of the National Bureau of Statistics (*https://data.stats.gov.cn/easyquery.htm?cn=C01*) and is presented in Table 1. These data are divided into two parts, the first part from the year 2010 to 2016 is used for modelling, and the second part from 2017 to 2019 is used for out-of-sample testing.

It is known that the whole modelling process can be established when system parameters b_1, b_2 and α are determined. Here the detailed modeling process of the CFDGM(1,1) model is shown. According to Table 1, the raw data on the area of drought disaster in China is

$X^{(0)}$=(13258.6, 16304.2, 9339.8, 1410.4, 12271.7, 10609.7, 9872.7, 9874.8, 7711.8, 7838.0).

Firstly, according to the Salp Swarm Algorithm and the raw data of the year from 2010 to 2016, we obtain the system nonlinear parameter α = 0.9188. Then on the base of Theorem 2, the values of B and Y can be given as follows.

$$B = \begin{pmatrix} 20964.7125 & -1 \\ 32942.3244 & -1 \\ 43513.7163 & -1 \\ 55198.0466 & -1 \\ 65169.3050 & -1 \\ 73971.2518 & -1 \end{pmatrix}, \quad Y = \begin{pmatrix} -15412.2249 \\ -8542.9990 \\ -12599.7846 \\ -10768.8760 \\ -9173.6408 \\ -8430.2527 \end{pmatrix}.$$

Table 1. The raw data of the area of drought disaster of China (thousand hectares)

Year	2010	2011	2012	2013	2014
Data	13258.6	16304.2	9339.8	14100.4	12271.7
Year	2015	2016	2017	2018	2019
Data	10609.7	9872.7	9874.8	7711.8	7838.0

It follows from formula $\left(b_1, b_2\right)^T = \left(B^T B\right)^{-1}\left(B^T Y\right)$ that b_1=0.0944, b_2=15413.6863. And then the expression of grey model CFDGM(1,1) can be written as

$$x^{(\alpha)}\left(k\right) = 0.9098^{k-1} x^{(\alpha)}\left(1\right) + 163207.9470\left[1 - 0.9098^{k-1}\right],\ k = 2,3,\ldots,$$

and the restored values of the CFDGM(1,1) model are easily computed. Actually, the other grey forecasting models DGM(1,1) and FDGM(1,1) can also be established. The computational results and errors are tabulated in Table 2 and Figure 1, and all results are reported to two decimal places.

It follows from Table 2 and Figure 1 that the three discrete grey models successfully catch the trend of the area of the drought disaster in China. The $MAPE_{simu}$, $MAPE_{pred}$ and $MAPE_{all}$ of the DGM(1,1) model are 12.54%, 6.87% and 10.65%, those of the FDGM(1,1) model are 12.37%, 6.19%, 10.31%, and those of the CFDGM(1,1) model are 12.31%, 6.18% and 10.27%, respectively. It is seen that the CFDGM(1,1) model obtains better results than the other discrete grey models. However, it must be admitted that their accuracy is basically the same, and they are all suitable for the area of drought disaster of China.

To further study the accuracy of the three grey discrete models, we consider the following two cases.

Case 1: The raw data from 2010 to 2015 are used for modelling, and the raw data from 2016 to 2019 are used for out-of-sample testing (r = 6).

Case 2: The raw data from 2010 to 2017 are used for modelling, and the raw data from 2018 to 2019 are used for out-of-sample testing (r = 8).

The computational results and corresponding errors are listed in Table 3 and Figure 2, and Table 4 and Figure 3, respectively.

It follows from Table 3 and Figure 2 that the $MAPE_{simu}$, $MAPE_{pred}$ and $MAPE_{all}$ of the DGM(1,1) model are 14.85%, 6.39% and 11.09%, those of the FDGM(1,1) model are 13.53%, 99.93%, 51.93%, and those of the CFDGM(1,1) model are 14.70%, 6.01% and 10.84%, respectively. It is obvious that the FDGM(1,1) model is inapplicable in the area of drought disasters in China.

It can be seen in Table 4 and Figure 3 that the $MAPE_{simu}$, $MAPE_{pred}$ and $MAPE_{all}$ of the DGM(1,1) model are 11.62%, 10.86% and 11.45%, those of the FDGM(1,1) model are 11.04%, 19.73%, 12.97%, and those of the CFDGM(1,1) model are 11.62%, 10.86% and 11.45%,

Table 2. The computational results of the DGM(1,1), FDGM(1,1) and CFDGM(1,1) models

Year	**Data**	**DGM(1,1)**	**APE**	**FDGM(1,1)**	**APE**	**CFDGM(1,1)**	**APE**
				fractional order=0.9185		*fractional order*=0.9188	
2010	13258.6	13258.60	0.00	13258.60	0.00	13258.60	0.00
2011	16304.2	14372.39	11.85	14231.56	12.71	14305.58	12.26
2012	9339.8	13366.88	43.12	13467.11	44.19	13450.93	44.02
2013	14100.4	12431.71	11.83	12555.11	10.96	12527.01	11.16
2014	12271.7	11561.97	5.78	11626.16	5.26	11605.59	5.43
2015	10609.7	10753.08	1.35	10725.76	1.09	10716.38	1.01
2016	9872.7	10000.78	1.30	9872.70	0.00	9872.70	0.00
2017	9874.8	9301.11	5.81	9074.57	8.10	9080.23	8.05
2018	7711.8	8650.39	12.17	8333.56	8.06	8340.70	8.16
2019	7838.0	8045.20	2.64	7649.10	2.41	7653.69	2.35
$MAPE_{simu}$			12.54		12.37		12.31
$MAPE_{pred}$			6.87		6.19		**6.18**
$MAPE_{all}$			10.65		10.31		10.27

Table 3. The computational results of the DGM(1,1), FDGM(1,1) and CFDGM(1,1) models (case 1)

Year	**Data**	**DGM(1,1)**	**APE**	**FDGM(1,1)**	**APE**	**CFDGM(1,1)**	**APE**
				fractional order=1.6157		*fractional order*=0.8579	
2010	13258.6	13258.60	0.00	13258.60	0.00	13258.60	0.00
2011	16304.2	14326.53	12.13	16304.20	0.00	14299.76	12.29
2012	9339.8	13361.47	43.06	11324.73	21.25	13520.58	44.76
2013	14100.4	12461.42	11.62	11114.39	21.18	12571.67	10.84
2014	12271.7	11622.00	5.29	11760.07	4.17	11582.65	5.62
2015	10609.7	10839.12	2.16	12842.73	21.05	10609.70	0.00
2016	9872.7	10108.98	2.39	14256.47	44.40	9679.65	1.96
2017	9874.8	9428.02	4.52	15976.41	61.79	8805.30	10.83
2018	7711.8	8792.94	14.02	18009.50	133.53	7992.02	3.63
2019	7838.0	8200.63	4.63	20379.47	160.01	7241.06	7.62
$MAPE_{simu}$			14.85		13.53		14.70
$MAPE_{pred}$			6.39		99.93		**6.01**
$MAPE_{all}$			11.09		51.93		10.84

Table 4. The computational results of the DGM(1,1), FDGM(1,1) and CFDGM(1,1) models (case 2)

Year	Data	DGM(1,1)	APE	FDGM(1,1)	APE	CFDGM(1,1)	APE
				fractional order=1.1497		*fractional order*=1.0000	
2010	13258.6	13258.60	0.00	13258.60	0.00	13258.60	0.00
2011	16304.2	14268.10	12.49	14767.29	9.43	14268.10	12.49
2012	9339.8	13339.12	42.82	13109.57	40.36	13339.12	42.82
2013	14100.4	12470.63	11.56	12124.66	14.01	12470.63	11.56
2014	12271.7	11658.68	5.00	11397.32	7.13	11658.68	5.00
2015	10609.7	10899.60	2.73	10809.91	1.89	10899.60	2.73
2016	9872.7	10189.94	3.21	10311.38	4.44	10189.94	3.21
2017	9874.8	9526.48	3.53	9874.80	0.00	9526.48	3.53
2018	7711.8	8906.22	15.49	9484.17	22.98	8906.22	15.49
2019	7838.0	8326.35	6.23	9129.22	16.47	8326.35	6.23
$MAPE_{simu}$			11.62		11.04		11.62
$MAPE_{pred}$			**10.86**		19.73		**10.86**
$MAPE_{all}$			11.45		12.97		11.45

respectively. These results show the FDGM(1,1) model is also inapplicable in the area of drought disasters in China. Moreover, we can see the optimal conformable fractional order of the CFDGM(1,1) model $\alpha = 1.0000$, and then it reduces to the classical DGM(1,1) model and has the same results in this case. It can also conclude that the CFDGM(1,1) model is suitable for the area of drought disasters in China.

4. Conclusion

In the current study, the area of China affected by the drought disaster is studied by using three discrete grey models, namely, the DGM(1,1) model, the FDGM(1,1) model, and the CFDGM(1,1) model. The optimal conformable fractional order is determined by the salp swarm algorithm. And we considered three different cases to study the accuracy of different grey models. The computational results show that the CFDGM(1,1) model performs better than the other discrete grey models, where the mean absolute prediction error $MAPE_{pred}$ in the three cases are 6.18%, 6.01%, and 10.86%, respectively. Moreover, it can be seen from Tables 2 – 4 and Figures 1 – 3 that the fractional order of the FDGM(1,1) model ranges from 0.9185 to 1.6157, and the conformable fractional order of the CFDGM(1,1) model ranges from 0.8579 to 1.0000. This means the CFDGM(1,1) model needs a narrower range of conformable fractional order and obtains higher accuracy results than the FDGM(1,1) in China's drought disaster.

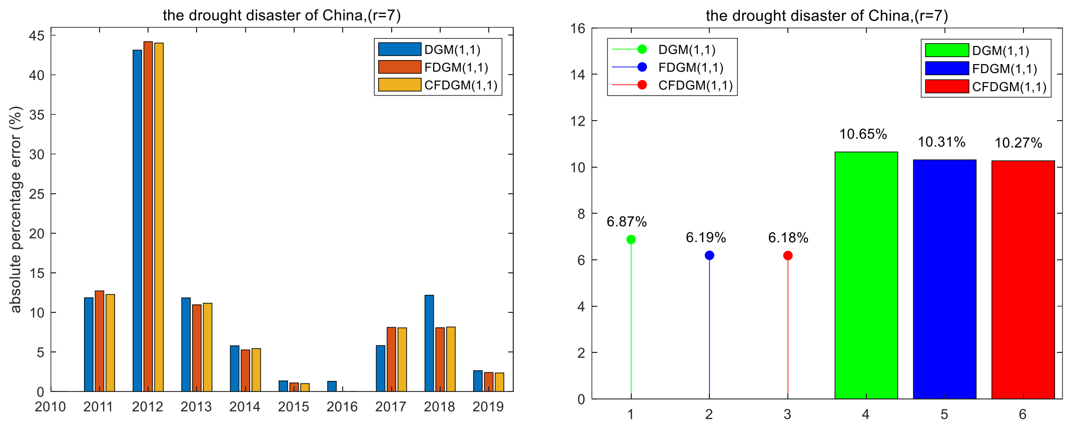

Fig 1. Results among the DGM(1,1), FDGM(1,1) and CFDGM(1,1) models

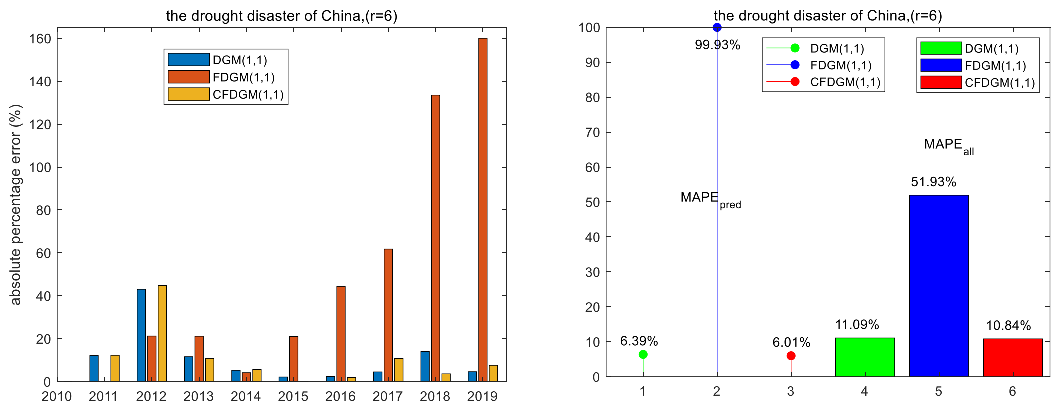

Fig 2. Results among the DGM(1,1), FDGM(1,1) and CFDGM(1,1) models (case 1)

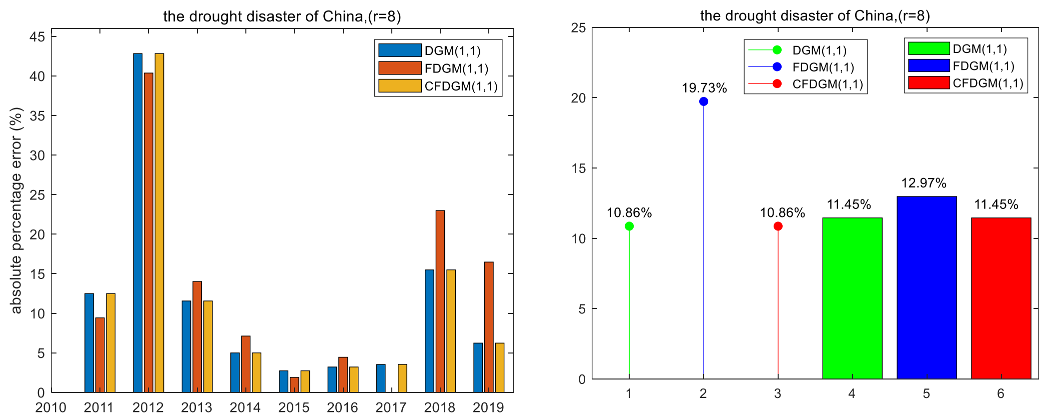

Fig 3. Results among the DGM(1,1), FDGM(1,1) and CFDGM(1,1) models (case 2)

Actually, the accuracy of the CFDGM(1,1) model has much room for improvement owing to the best $MAPE_{pred}$ is 6.01% in the area of drought disaster in China. On the one hand, the raw data of China's drought disaster exhibits considerable variation, while the CFDGM(1,1) model is a linear model and cannot depict the data's fluctuation features. On the other hand, it is infeasible to consider other factors, such as population, and meteorological, which affect the current situation of China's drought disaster in the newly proposed model owing to the CFDGM(1,1) model is univariate. In the future, some nonlinear grey forecasting models or multivariate grey forecasting models with conformable fractional order can be considered and applied to the area of drought disaster in China.

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