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Reliability Evaluation Model for Complex Equipment Fusing General Uncertainty Variables

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Abstract: Due to the uncertainty of cognition and the difficulty of obtaining information, the reliability information of complex equipment is full of uncertainty. In order to make full use of multi-source uncertain reliability information, a reliability evaluation model based on D-S evidence theory and general uncertainty theory is proposed. The main work is as follows: Firstly, the basic probability assignment of evidence theory is carried out through general uncertainty theory for random, fuzzy, grey and rough reliability data in the development process of complex equipment. Second, in order to address the fusion of conflicting evidence, the weights of the evidences to be fused are corrected from three different perspectives. On this basis, the optimal weight combination is obtained by the TOPSIS method. The mission reliability evaluation result of the whole complex equipment is obtained by using the Dempster combination rule; Finally, an arithmetic example illustrates that the method proposed in this study is characterized by more conservative assessment results and more accurate reflection of changes in reliability confidence.

Keywords: Grey linguistics; D-S evidence theory; general uncertainty theory; complex equipment; reliability evaluation

1. Introduction

Under the guidance of China's strategy for building a manufacturing powerhouse, research on the reliability issues of high-end and complex equipment manufacturing represented by aerospace rockets, space shuttles and the like is particularly important. However, such equipment often has the characteristics of high-reliability products, that is, small samples of product failure data, high costs for reliability tests, etc. Meanwhile, due to reasons such as the limitations of human cognition and the complexity of structures, there is a large amount of reliability information with uncertainties, which have brought many difficulties to the reliability evaluation of complex equipment. How to comprehensively utilize the reliability-related information in all aspects of complex equipment from design to application has become a hot topic in current research. Currently, the reliability evaluation models related to the fusion of multi-source information are mainly developed from two aspects. One is to utilize Bayesian networks (Li *et al.*, 2022; Songshi *et al.*, 2019), and the other is to utilize the D-S evidence theory (Rui *et al.*, 2016; Yu *et al.*, 2021). Among them, Bayesian networks can only be used for the fusion and inference of prior information with precise information. However, a large amount of information in the development process of complex equipment is imprecise. The D-S evidence theory can integrate various types of uncertain data and is therefore more suitable for the reliability evaluation work of complex equipment (Tangfan, 2022).

In recent years, the evidence theory has been deeply applied in research on the reliability evaluation of high-reliability products (Rongxi *et al.*, 2018; Yanjun *et al.*, 2024; Qingde, 2017; Xiaojie, 2024; Sirui *et al.*, 2023), the safety evaluation of complex products (Taiping & Zhong, 2023; Juan, 2021; Yu & Xiaochun, 2021), fault diagnosis (Jiahang *et al.*, 2023; Zhiwei & Le, 2023; Yongqiang, 2022; Jie *et al.*, 2023; Zhongqiang *et al.*, 2022) and other aspects. However, since the classic Dempster combination rule will produce results contrary to the facts when fusing highly conflicting evidence (Zadeh, 1984), the fusion processing of conflicting evidence is often involved in these studies. To solve this problem, one idea is to modify the original combination rule (Cong *et al.*, 2019; Ghosh *et al.*, 2022; Khan & Koo, 2015; Pai & Gaonkar, 2020), and the other is to modify the original evidence through a discount coefficient (Djapic *et al.*, 2012; Farmer, 2017; Sezer *et al.*, 2022). Some scholars have pointed out that modifying the original combination rule will make it lose many excellent properties (Zhan, 2024), so current research mainly focuses on the modification of the evidence body. The processing of the discount coefficient is mainly reflected in the allocation of the weights of the evidence body. At present, most of the research on the allocation of the weights of the evidence body focuses on the characteristics of the data itself and seldom considers the role of subjective evaluation in the weights (Yanjun *et al.*, 2024; Shiyuan, 2024; Fan, 2023). There may be a certain coupling relationship among the reliability data of complex equipment, and it is difficult to reflect this effect only by assigning values from an objective perspective.

Meanwhile, how to represent and process multi-source uncertain reliability information is also a key issue. Previous studies mainly focused on the fusion of reliability information with a certain probability distribution (Rongxi *et al.*, 2018), or directly analyzed the results of expert evaluations using the central limit theorem (Sirui *et al.*, 2023). Liu proposed the general uncertainty theory in 2022 (Liu & Tang, 2023), which can process data with random, fuzzy, grey and rough characteristics simultaneously and can handle the reliability information in the development process of complex equipment more comprehensively.

Based on this, in order to better integrate various types of reliability data with uncertainties and considering the importance of subjective weighting in the reliability engineering of complex equipment, this study first starts from the general uncertainty theory and explores the conversion methods of the basic probability assignment function in the evidence theory for different types of uncertain data. Subsequently, with the help of the concepts of the core and uncertainty in the general uncertainty theory, the weights of the evidence are allocated from three aspects: the similarity among the evidence, the uncertainty of the evidence itself and the subjective evaluation. Innovatively, the Shapley value method is used for the processing of subjective weights. On this basis, the comprehensive optimal weights are obtained by using the technique for order preference by similarity to an ideal solution (TOPSIS), and the probability assignment is carried out at the decision-making level of the evidence theory to obtain the final evaluation results. Finally, the feasibility and effectiveness of the method proposed in this study are illustrated through a small case.

2. Theoretical Basis

2.1 D-S Evidence Theory

DEFINITION 1 (Dempster, 2008): *Frame of Discernment*. In the evidence theory, the set of all possible objects under study is called the frame of discernment, denoted as Θ, the representation way is $\Theta=\{\theta_1,\theta_2,...,\theta_n\}$, where $\theta_i, i=1,2,..,n$ represents independent and mutually exclusive elements, n represents the number of elements.

DEFINITION 2 (Dempster, 2008): *Basic Probability Assignment*. The power set of the frame of discernment 2^{Θ} represents any arbitrary combination of elements in Θ. The basic probability assignment function is a mapping from the power set to the interval [0,1], also known as the mass function, which satisfies the following conditions

$$\begin{cases} \sum_{A \subseteq \Theta} m(A) = 1 \\ m(\varnothing) = 0 \end{cases} \tag{1}$$

Among them: represents a basic element in the power set 2^{Θ} ; $m(A)$ represents the corresponding Basic Probability Assignment (BPA) or mass function value of A, also known as the body of evidence, and its meaning is the direct support degree of the evidence information for the proposition A. If $\mathrm{m(A)}>0$, then A is called a focal element of the BPA.

In practical problems, due to different perspectives of thinking about problems and different sources of information collection, different Basic Probability Assignments (BPAs) may be obtained for the same problem. These are called different pieces of evidence. In order to comprehensively utilize all the evidence information to obtain more reliable conclusions, it is necessary to fuse different pieces of evidence information.

DEFINITION 3 (Shafer, 1976): *Dempster's combination rule*. Dempster's combination rule is the most classic combination rule in evidence theory and has some excellent properties. Under the same frame of discernment Θ, suppose m_1, m_2 are two independent pieces of evidence, and their focal elements are $A_1, A_2, ..., A_p$ and $B_1, B_2, ..., B_q$ respectively. Then, according to Dempster's combination rule, the combined evidence of and is as follows:

$$m(A) = m_1(A) \oplus m_2(B) = \begin{cases} \dfrac{\sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j)}{1-k}, & A \neq \varnothing \\ 0, & A = \varnothing \end{cases} \tag{2}$$

Among them: $i = 1, 2, ..., p$; $j = 1, 2, ..., q$; $k = \sum_{A_i \cap B_j = \varnothing} m_1(A_i) m_2(B_j)$ is the conflict factor, which reflects the degree of conflict between the two pieces of evidence. Similarly, for n pieces of evidence under the same frame of discernment, the combination formula under Dempster's combination rule is as follows:

$$m_{1\oplus 2\oplus ...\oplus n}(A) = \frac{\sum_{A_1 \cap A_2 \cap ... \cap A_n = A} m_1(A_1) m_2(A_2) ... m_n(A_n)}{1 - \sum_{A_1 \cap A_2 \cap ... \cap A_n = \varnothing} m_1(A_1) m_2(A_2) ... m_n(A_n)} \tag{3}$$

According to the Transferable Belief Model (TBM) proposed by Smets, when using the D-S evidence theory to make decisions, it is necessary to go through a two-layer fusion structure, namely the Credal layer and the Decision (Pignistic) layer (Smets & Kennes, 1994). The initial belief function is transferred at the Credal layer to construct a reasonable initial Basic Probability Assignment (BPA). And when it comes to making a decision, it is required to use the probability transformation method to obtain the probability at the Pignistic layer, also known as the revised BPA, and then make the final decision based on the probability distribution at the Pignistic layer.

DEFINITION 4 (Smets & Kennes, 1994): *Pignistic Probability Transform*. The probability distribution obtained after the pignistic probability transform (PPT) is denoted as $BetP$. The specific formula is as follows:

$$BetP(A) = \sum_{B \subset \Theta} m(B) \frac{|A \cap B|}{|B|} \tag{4}$$

Among them: $BetP$ is also called the betting probability, and $|A|$ is the cardinality of A. It can be seen from the calculation formula that the essence of the PPT is to distribute the belief corresponding to the multi-proposition focal elements to each individual proposition focal element in an evenly divided manner.

2.2 General Uncertainty Theory

The reliability data of complex equipment are usually complex data with the characteristics of uncertainty information such as randomness, fuzziness, greyness, roughness, etc. It is difficult to conduct comprehensive and effective analysis on these complex data by using any single analysis method among probability statistics, fuzzy mathematics, grey system theory or rough set theory.

DEFINITION 5 (Liu & Tang, 2023): *General Uncertainty Data.* The mixed uncertainty data with the characteristics of uncertainty information such as randomness, fuzziness, greyness, and roughness are called general uncertainty data (GUD).

DEFINITION 6 (Liu & Tang, 2023): *General Uncertainty Variable.* A set composed of random variables, fuzzy numbers, grey numbers and interval rough numbers is called a set of general uncertainty variables (GUVs). GUVs can be divided into continuous GUVs and discrete GUVs.

Liu proposed that the operation methods of General Uncertainty Variables (GUVs) can be divided into two types. One is the operation of interval GUVs based on the coverage of possible values, which is similar to interval operations; the other is the operation based on the kernel and the degree of uncertainty. Liu also gave the rules for operations based on the kernel and the degree of uncertainty (Liu & Tang, 2023). This research will follow this part of the calculation rules.

DEFINITION 7 (Liu & Tang, 2023). *Kernel of the General Uncertainty Variable.* The average value of all possible values of a General Uncertainty Variable (GUV) is called the kernel of the GUV.

For a specific General Uncertainty Variable (GUV) with value distribution information, the calculation rules for the kernel are as follows: For a GUV that was originally a random variable, the value of its kernel can be taken as its mathematical expectation; for a GUV that was originally a fuzzy number, its kernel is the point with the largest membership degree; for a GUV that was originally an interval grey number, its kernel is the midpoint of the interval; for a GUV that was originally an interval rough number, its kernel is the midpoint of the lower approximation.

DEFINITION 8 (Liu & Tang, 2023): *Degree of uncertainty of General Uncertainty Variable.* Suppose the background that generates the GUV is Ω, and μ is the measure of Ω. Then the calculation formula for the degree of uncertainty of the GUV (abbreviated as u°) is as follows:

$$u^{\circ}(GUV)=\frac{\mu(GUV)}{\mu(\Omega)} \tag{5}$$

It satisfies the property of normality in its definition.

The degree of uncertainty of a GUV reflects the degree of uncertainty of the thing described by the GUV. For a completely certain and real number, its degree of uncertainty is 0. If the value range of a GUV is completely unknown or in the case where the value range of the GUV is equal to its background Ω, then the degree of uncertainty of this GUV is 1. For most interval GUVs, their degrees of uncertainty are between 0 and 1. The closer u° is to 0, the smaller the uncertainty of the GUV values. Conversely, the closer u° is to 1, the greater the uncertainty of the GUV values.

For convenience, we can denote the GUV as $x, y, z, \cdots$, and denote the corresponding kernel as $\hat{x}, \hat{y}, \hat{z} \cdots$.

DEFINITION 9 (Liu & Tang, 2023). *Simplified form of General Uncertainty Variable.* Let x be a GUV, $\hat{x}$ be the kernel of x, and u° be the degree of uncertainty of x. Then the simplified form of is $\hat{x}_{(u^{\circ})}$.

3. Reliability Evaluation Model for Fusing General Uncertainty Variables

Suppose there are n initial information sources, and the data types include random, fuzzy, grey, and rough. According to the Transferable Belief Model (TBM), the modeling steps of the reliability assessment model for fusing general uncertainty variables are as follows: Firstly, it is necessary to determine the frame of discernment and select the reliability assessment indicators. Secondly, according to the data types of the information sources, the reliability data of the same type should be fused, and the initial Basic Probability Assignment (BPA) should be calculated. Subsequently, the weight distribution of the BPA needs to be revised, and this part is mainly considered from both objective and subjective aspects. Finally, the Probability Projection Transformation (PPT) is carried out to obtain the final decision probability. The flowchart of the assessment model in this research is shown in Figure 1.

3.1 Determination of the Frame of Discernment for Reliability Evaluation

The reliability evaluation indicators for complex equipment include reliability (probability measure), Mean Time Between Failure (MTBF), Mean Time To Repair (MTTR), availability and so on. For indicators such as MTBF, MTTR and availability, they are all obtained by collecting the time intervals of fault occurrences or maintenance activities. This kind of data cannot reflect the characteristics of General Uncertainty Data (GUD). Therefore, it is more suitable for the content of this research to select an indicator that can comprehensively evaluate the ability of complex

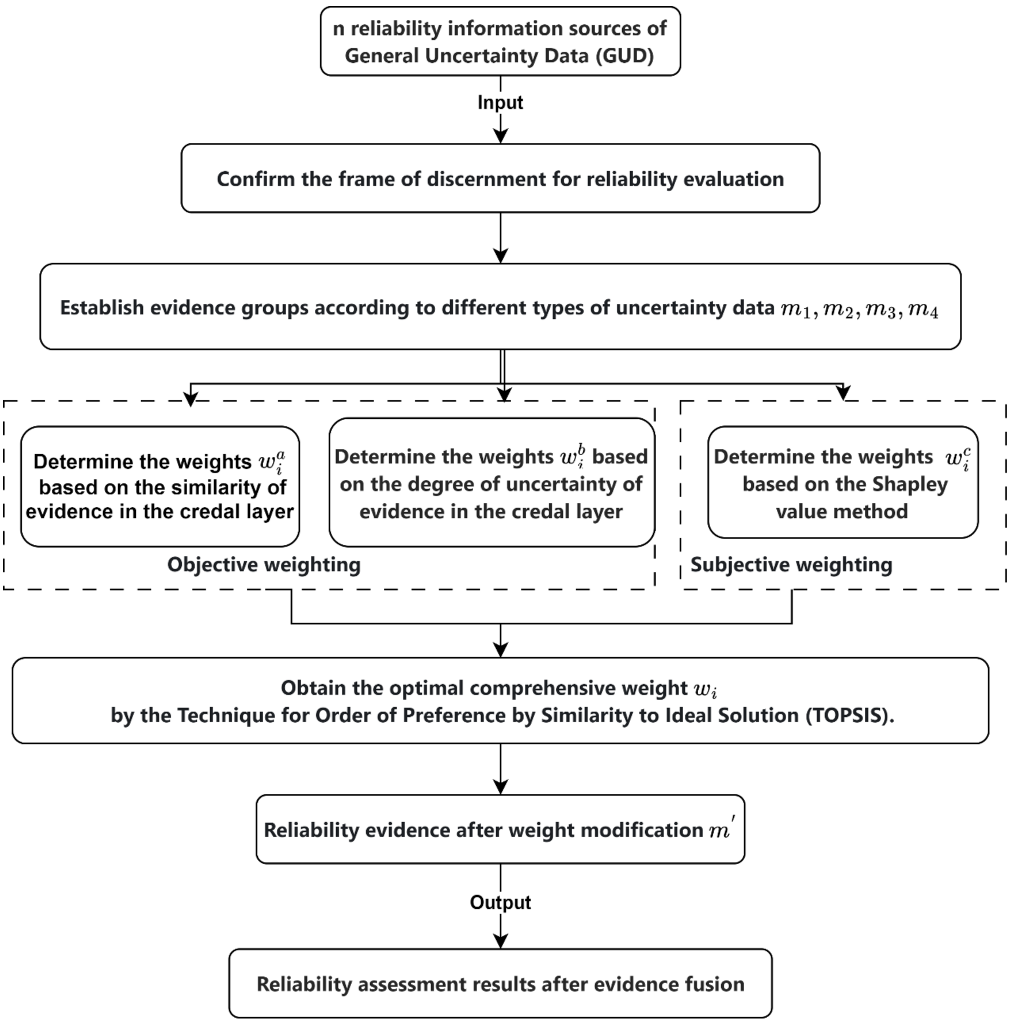

Figure 1. Flowchart of the evaluation model for complex equipment with fused general uncertainty variables

equipment to complete specified tasks under specified conditions. Reliability (probability measure) refers to the probability that the equipment can complete the specified functions under the specified conditions and within the specified time. The reliability value can be obtained through probability distributions, reliability block diagrams, reliability tests and other means, and it can better support the fusion evaluation of multi-source information. This research selects reliability (probability measure) as the content of the frame of discernment, unless otherwise specified in the following text, "reliability" can be regarded as the probability measurement of reliability.

Since reliability is a probability value within a certain range $[0,1]$, the frame of discernment is thus determined as $\Theta = \{[0,1]\}$ accordingly. According to the principle of least commitment proposed by Smets in the Transferable Belief Model (TBM) (Smets & Kennes, 1994), under the confidence interval $[r_L, r_U]$ of the complex equipment's reliability with a certain confidence level $1-\alpha$, the evidence $m([0,1])$, $m([r_L, r_U])$ transformed from the information sources can be calculated according to the following formulas:

$$\begin{cases} m([0,1]) \times \dfrac{r_U - r_L}{1-0} + m([r_L, r_U]) = 1-\alpha \\ m([0,1]) + m([r_L, r_U]) = 1 \end{cases} \tag{6}$$

3.2 Evidence Modeling for Different Reliability Information Sources

The reliability information of complex equipment comes from a wide variety of information sources, such as failure data, expert evaluations, key equipment parameters, reliability test results of subsystems or critical components, etc. These pieces of information may possess uncertainty characteristics of multiple types, such as randomness, fuzziness, greyness, roughness, and so on. Therefore, all the reliability information sources of complex equipment can be analyzed as GUVs. Classify and analyze the GUVs according to different uncertainty types. Internally, relatively consistent analysis tools can be used for analysis, and externally, the overall importance of this type of data can be evaluated. Meanwhile, within the reliability data of the same uncertainty type, calculations such as the fusion of data at the same level and the transfer of data at different levels may also occur according to the levels in the reliability block diagram. Consequently, the reliability evaluation of complex equipment itself has the characteristic of being multi-level. However, the information fusion processes at different levels have similar procedures, and conducting uncertainty transfer analysis through a complete reliability block diagram will further expand the scale of this research. Therefore, to highlight the key points of this research, it is assumed in the study that data of the same uncertainty information type can be fused by a simple averaging method, and the result after fusion reflects the overall reliability characteristics of the complex equipment. Appropriate adjustments can be made in practical applications.

The following will introduce the BPA of GUVs with different uncertainty types.

3.2.1 Failure data of the random variable type. A random variable is a concept from probability and statistics. Given a sample space Ω, if for any given sample point ω, there exists a unique real number X corresponding to ω, and X can take different values with a certain probability, then X is called a random variable. Random variables have probability distribution functions. For example, in the actual engineering process, a large amount of failure data for the components of complex equipment will be collected - it may be the time of failure occurrence or the time between failures, and these data conform to a certain probability distribution. Engineers summarize them in reliability manuals and they are widely used in reliability engineering.

In the probability distribution function of a random variable, the confidence level refers to the probability that a sampling point falls within the corresponding confidence interval. According to the general uncertainty theory, the confidence interval can be regarded as the upper and lower limits of the reliability estimate, and the confidence level can be regarded as the Pignistic probability of the confidence interval. Different types of components may have different probability distribution functions, but all of them can obtain the confidence interval $[r_L, r_U]$ and the point estimate $\hat{r}$ of reliability through the transformation of probability theory formulas under the premise of the known confidence level α and probability distribution $f(x)$. The specific formulas are as follows:

$$\begin{cases} \hat{r} = \dfrac{r_L + r_U}{2} \\ \int_{r_L}^{r_U} f(x)dx = 1-\alpha \end{cases} \tag{7}$$

According to formula (6), the BPA of the reliability information of the random variable type can be obtained.

3.2.2 Expert evaluations of the fuzzy set type. A fuzzy set is a tool that can describe the ambiguous membership relationship of a set. In classical set theory, an element either belongs to a certain set or does not belong to it, and the boundary is very clear. However, a fuzzy set describes the degree to which the domain of discourse U belongs to a set A in the form of a membership function $\mu_A : U \to [0,1]$. The closer the value of μ_A is to 1, the higher the degree to which it belongs to the set A, and thus it can better describe the uncertainty of human cognition in real life. In the field of reliability engineering, experts may often not be able to give an exact value when describing the mission reliability of a complex system as a whole. Therefore, using fuzzy sets can better model and analyze such expert evaluation information.

Currently, there are many tools for describing fuzzy linguistic variables, including the Uncertain Linguistic Term set (ULTs), the Hesitant Fuzzy Linguistic Term set (HFLTs), the Extend Hesitant Fuzzy Linguistic Term set (EHFLTs), and so on. Du proposed a Grey Linguistic Term set (GLTs), proved that ULTs, HFLTs, and EHFLTs are degenerate patterns of GLTs under certain circumstances, and demonstrated that GLTs can describe fuzzy linguistic variables with more general properties (Du *et al.*, 2023). Du also put forward the concepts of the kernel and the greyness for the operations of GLTs. The research on the BPA of fuzzy linguistic variables in this study is based on this.

DEFINITION 10 (Du *et al.*, 2023): *Grey Linguistic Term set.* Given a continuous linguistic term set $\bar{S} = \{s_i \mid s_0 \le s_\alpha \le s_t, \alpha \in [0,t]\}$, then an ordered union of closed or open linguistic intervals $G_{\bar{S}}(x) = \{s_i \mid s_i = [s_i^-, s_i^+] \subseteq \bar{S}\}$ is called a Grey Linguistic Term set (GLTs). Among them: $i = 1, 2, ..., n$, where n is an integer greater than 0, and $s_i^-, s_i^+ \in \bar{S}$, $s_{i-1}^+ \le s_i^- \le s_i^+ \le s_{i+1}^-$. In addition, the lower and upper bounds of $G_{\bar{S}}(x)$ are defined as $s^- = \inf_{s_i^- \in G_{\bar{S}}(x)} s_i^-$ and $s^+ = \sup_{s_i^+ \in G_{\bar{S}}(x)} s_i^+$, respectively. The concise form of GLTs is $G_{\bar{S}}(x) = \bigcup_{i=1}^{n} [s_i^-, s_i^+]$.

DEFINITION 11 (Du *et al.*, 2023): *Greyness of Grey Linguistic Term set.* For a given Grey Linguistic Term set, $s_{\min} = \min\{\bar{S}\} = s_0$, $s_{\max} = \max\{\bar{S}\} = s_t$, its greyness is defined as:

$$g^{\circ}(G_{\bar{S}}(x)) = \frac{\Delta^{-1}(s^+) - \Delta^{-1}(s^-)}{\Delta^{-1}(s_{\max}) - \Delta^{-1}(s_{\min})} \tag{8}$$

Among them: Δ^{-1} is the inverse transformation function of the fuzzy language representation model.

DEFINITION 12 (Du *et al.*, 2023). *Kernel of Grey Linguistic Term set.* For a given GLTs, let the kernel of the linguistic interval $[s_i^-, s_i^+]$ be $\hat{s}_i = \Delta((\Delta^{-1}(s_i^-) + \Delta^{-1}(s_i^+))/2) = (s_i, \alpha)$, then the kernel of GLTs is defined as:

$$\hat{G}_{\bar{S}}(x) = \Delta(\frac{\sum_{i=1}^{n} \Delta^{-1}(\hat{s}_i)}{n}) \tag{9}$$

The greyness and the kernel contain all the information about GLTs. According to the general uncertainty theory, through simple numerical transformations, $\Delta^{-1}[s_i^-, s_i^+]$ can be converted into $[r_L, r_U]$, and the concept of the greyness of GLTs is similar to that of the uncertainty. Let it be equal to α, so that the BPA of the reliability information of the fuzzy set type can be obtained according to formula (6).

3.2.3 Information on key equipment parameters of the grey number type. A grey number is used to describe a number whose exact value is unknown while only its value range is known. Usually, a grey number is represented by $\otimes \in [a,b]$, where the true value of the grey number is taken within $[a,b]$, but it is unclear which specific value it is. In the practice of reliability engineering, the values of some important parameters can sometimes directly reflect the reliability level of complex equipment or subsystems. However, due to the incompleteness of information collection and the uncertainty of cognition, an exact value is often unavailable. In engineering practice, through the collection of historical data and the judgment based on experts' experience, the values of key parameters can be determined within a rough range.

Generalized Grey Numbers have properties similar to those of GUVs. They also possess the concepts of kernel and greyness, and their calculation processes are analogous. You can refer to the previous content (Liu *et al.*, 2004). According to the general uncertainty theory, after a series of Generalized Grey Numbers are combined in accordance with the rules set by Liu and Tang (2023) and then normalized into standard grey numbers with a universe of discourse of 1, the simplified form of the final result $\hat{\otimes}_{(g^\circ)}$ can be obtained as $\hat{\otimes}_{(g^\circ)}$, where $\hat{\otimes}, g^\circ \in [0,1]$. The concept of the greyness of a grey number is similar to that of the uncertainty degree u° of a GUV. The kernel is a real number that can represent the grey number, and the greyness is the ratio of the value range generated by the grey number to its background domain (Sifeng *et al.*, 2004). Its definition satisfies the normality, so the confidence level α can still be used for equivalence. Then, based on formula (6), the BPA of the reliability information of the grey number type can be obtained.

3.2.4 Reliability test results of the rough set type. A rough set uses the method of upper and lower approximations to describe unknown information with known information. Assume that $IRN(I_i) = (RN(I_{li}), RN(I_{ui}))$, where $RN(I_{li}) = [c,d]$, $RN(I_{ui}) = [a,b]$, and $a \le c \le d \le b$. That is, if both the lower approximation and the upper approximation of a rough variable are intervals, then $IRN(Ii)$ is called an interval rough number. The analysis tools for reliability assessment and reliability growth are often used in combination. Because in different reliability growth stages, the data from reliability assessment can help establish the analysis models for reliability growth, such as the Duane model and the AMSAA model. In the same reliability growth stage, the analysis results of the reliability growth model can also help assess reliability in return. During the development process of complex equipment, reliability tests will be carried out on subsystems and key components. The test results can reflect the overall reliability of the equipment to a certain extent. Meanwhile, the reliability growth data of historical or similar products can also be used as a reference for assessment. These reliability estimates can form the upper and lower approximations of the true value of the reliability assessment of complex equipment. At the same time, due to reasons such as the incomplete reflection of the mission system and the inability to accurately determine the similarity coefficient, the upper and lower approximations themselves may also be interval values, which constitutes a rough set problem.

As a tool for studying uncertain knowledge, rough sets are widely used in fields such as attribute reduction, knowledge reasoning, and information judgment. To avoid further expanding the scope of this study, it is assumed here that the values obtained by rough sets are already the results after attribute reduction. In fact, interval rough numbers are similar to trapezoidal fuzzy numbers, so simplified calculations can be carried out.

Assume that $RN\left(I_{li}\right)=\left[c,d\right]$ is the value of the lower approximation of the reliability obtained through the reliability tests of components, and $RN\left(I_{ui}\right)=\left[a,b\right]$ is the value of the upper approximation of the reliability obtained from the reliability growth data of historical products or similar products, where $0\le a\le c\le d\le b\le 1$. Then, according to the general uncertainty theory, it can be considered that the point estimate of the reliability is $\hat{r}=(c+d)/2$, and the confidence level is equal to the uncertainty degree. The calculation formula is $\alpha=u^{\circ}=[\mu([a,b])-\mu([c,d])]/\mu(\Omega)$. Based on this, the BPA of the reliability information of the rough set type can be obtained.

3.3 Comprehensive weight modification of evidence bodies considering subjective and objective factors.

3.3.1 The similarity among the evidences in the Credal layer. In order to avoid the impact of interfering evidences on the overall evidences, it is necessary to analyze the similarity degree among the evidences from the Credal layer. Through the results of pairwise comparisons among the evidences, it is judged which one or several evidences have the greatest differences from the others, and then the weights of these interfering evidences are reduced. In the evidence theory, the measures for evaluating the similarity among evidences are often judged by distance indicators, and among them, the Jousselme distance is the most widely used method (Jousselme *et al.*, 2001). This study adopts the Jousselme distance to measure the similarity among the evidences in the Credal layer.

DEFINITION 13 (Jousselme *et al.*, 2001): *Jousselme Distance*. Let the frame of discernment be $\Theta=\{\theta_1,\theta_2,...,\theta_n\}$, the power set be $2^{\Theta}=\{A_1,A_2,...,A_{2^n}\}$, and the BPAs corresponding to n pieces of evidence be $m_1,m_2,...,m_n$. Then the Jousselme distance between the evidences is defined as follows:

$$d_{ij}=\sqrt{\frac{1}{2}(M_i-M_j)^T D(M_i-M_j))} \tag{10}$$

Among them: $M_i=[m_i(A_1),m_i(A_2),...,m_i(A_{2^n})]^T$; D is a $2^n\times 2^n$ positive definite matrix, where $D_{ij}=|A_i\cap A_j|/|A_i\cup A_j|$; $|\bullet|$ represents the cardinality of the set.

The support degree of each piece of evidence obtained from the Jousselme distance is:

$$Sup(m_i)=\frac{\min_i d_i}{d_i},\quad i=1,2,..,n \tag{11}$$

Among them: $d_i=\sum_j d_{ij}$ is the sum of the distances between evidence m_i and other evidences, which reflects the total difference from the other $n-1$ pieces of evidence. Thus, the similarity weights of each piece of evidence can be obtained as follows:

$$w_i^a=\frac{Sup(m_i)}{\sum_{j=1}^{n}Sup(m_j)} \tag{12}$$

3.3.2 The uncertainty of the evidences themselves in the Credal layer. Besides the identification of interfering evidences, the uncertainty of different evidences themselves is also an aspect that affects the credibility of the evidences. Due to differences in cognition and varying degrees of difficulty in data acquisition, the results of the uncertainty degree u° among different evidences may vary greatly. The smaller the uncertainty degree of an evidence is, the more reliable the evidence it represents. Therefore, a higher weight should be assigned when distributing weights.

Use the reciprocal of the uncertainty degree of different evidences as the basis for the allocation of weights based on the uncertainty of the evidences themselves. Let u° represent the uncertainty degree of evidence m_i, then the calculation formula for the uncertainty weight is as follows:

$$w_i^b = \frac{1/u_i^{\circ}}{\sum_{j=1}^{n} 1/u_j^{\circ}} \tag{13}$$

3.3.3 The Shapley value method for subjective evaluation. After assigning weights to the objective characteristics of the evidences in the Credal layer, it is necessary to modify the weights among the evidences again from a subjective perspective. Because in the fusion process of the D-S evidence theory, it is required to ensure that the evidences are independent of each other and do not influence one another. However, the multi-source reliability information of complex equipment may not meet the requirement of mutual independence. For example, the results of expert evaluations may come from failure data, estimations of important parameters, and reliability tests. Meanwhile, in this study, different evidences represent reliability data of different uncertainty types, and the reliability data under the same type may jointly reflect the characteristics of a certain subsystem. The Shapley value method can reflect the emergence effect, coupling effect, etc. among different subsystems. For instance, the results obtained by using failure data or expert evaluations alone may not be reliable, but when combining failure data and expert evaluations simultaneously, the results will become more reliable.

The Shapley value method is an effective approach used to solve the problem of profit distribution in cooperative games. It aims to determine the fair share that each participant should receive based on the marginal contributions of each participant to different coalition combinations. The degree of support provided by the combination of evidences for the reliability assessment can be regarded as the contribution of the coalition, and thus the fair share of each evidence, that is, the absolute weight of the evidence, can be obtained. The following will briefly introduce the solution process of the Shapley value method.

DEFINITION 14: *The Shapley value method.* Let the frame of discernment be $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, the power set be $2^{\Theta} = \{A_1, A_2, ..., A_{2^n}\}$, and the BPAs corresponding to n pieces of evidence be $m_1, m_2, ..., m_n$. Regard $m_1, m_2, ..., m_n$ as the n members within the cooperative game system, denoted by $N = \{m_1, m_2, ..., m_n\}$. Let S be different coalitions composed of different members, $v(S)$ be the payoff of coalition S, and the marginal contribution created by member m_i for its own coalition when participating in different coalitions S is denoted as $v(S) - v(S\backslash\{\mathrm{m_i}\})$. Then the fair share of benefits that member m_i obtains from the overall benefits $v(N)$ is:

$$\varphi\_\mathrm{m}_i(v) = \sum_{S \in N} \frac{[(|S|-1)!(n-|S|)!]}{n!} \times [v(S) - v(S\backslash\{\mathrm{m_i}\})] \tag{14}$$

Thus, the subjective weight is obtained as follows:

$$w_i^c = \frac{\varphi\_\mathrm{m_i}(v)}{\sum_{j=1}^{n} \varphi\_\mathrm{m_j}(v)} \tag{15}$$

3.3.4 Optimal comprehensive weight. After the weight assignments from both subjective and objective aspects, it is necessary to comprehensively consider the above three types of weights. The optimal weights can be obtained through a simple weighted method. To obtain a more reasonable weight distribution, this study adopts the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) for calculation.

Taking the BPAs of 4 pieces of evidence as an example. First, construct a decision matrix A with the three weight sequences as rows, that is:

$$A = \begin{pmatrix} w^a \\ w^b \\ w^c \end{pmatrix} = \begin{pmatrix} w_{11}, w_{12}, w_{13}, w_{14} \\ w_{21}, w_{22}, w_{23}, w_{24} \\ w_{31}, w_{32}, w_{33}, w_{34} \end{pmatrix} \tag{16}$$

Let $A_j^+ = \max_i(w_{ij})$, $A_j^- = \min_i(w_{ij})$, $i = 1,2,3; j = 1,2,3,4$, Then we have $d_i^+ = \sqrt{\sum_{j=1}^4 (w_{ij} - A_j^+)^2}$, $d_i^- = \sqrt{\sum_{j=1}^4 (w_{ij} - A_j^-)^2}$. The closeness of each weight sequence to the ideal solution is: $C_i = d_i^- / (d_i^+ + d_i^-), i = 1,2,3$. Finally, the comprehensive optimal weight is obtained as:

$$w_j = \sum_{i=1}^{3} C_i w_{ij} \tag{17}$$

4. Case Study

A certain type of space carrier rocket is about to carry out a launch mission. In order to ensure the smooth progress of the mission, it is necessary to predict the overall mission reliability of the carrier rocket before the start of the mission. In order to obtain as reliable a result as possible within the shortest possible time, the expert group of this model has decided to comprehensively consider multiple different reliability information sources to evaluate the mission reliability level of the carrier rocket.

The judgment requirements of the expert group are as follows: (1) The point estimate of reliability should not be lower than 96%; (2) The confidence level should be greater than 90%. That is, the simplified form of the calculated value $\hat{r}_{(u^\circ)}$ of the reliability of a certain type of carrier rocket should meet the following conditions:

$$\begin{cases} \hat{r} \geq 0.96 \\ u^\circ \leq 0.1 \end{cases}$$

Through the efforts of the team members, four types of reliability data were finally obtained: (1) Failure data; (2) Judgments made by experts based on their experience; (3) Numerical information on important parameters; (4) Reliability predictions of historical products and similar products, as well as the reliability test results of subsystems and key components.

According to the historical data and by referring to the engineer's manual, it is judged that the failure data conforms to the normal distribution of $R_E \sim N(0.97, 0.006)$.

Set $\{s_1, s_2, s_3, s_4\}$ as the language assessment level, which respectively corresponds to {Poor, Medium, Good, Excellent}, and the corresponding reliability is divided into $\{[-,0.85],[0.85,0.90],[0.90,0.95],[0.95,1.00]\}$. The result obtained by using GLTs to perform Union operators for the evaluation results made by the expert group based on their experience is: $(s_4, -0.7)_{(0.06)}$.

The result of simplifying the numerical information of the important parameters into standard grey numbers is: $\hat{\otimes}_{(g^\circ)} = 0.96_{(0.08)}$.

After the reliability test results are converted into reliability, they exhibit the characteristics of rough sets: $[[0.92, 0.98],[0.9,1.0]]$.

4.1 Establish the Frame of Discernment and BPA.

Let the frame of discernment be reliability (probability measure), that is, $\Theta \in \{[0,1]\}$.

(1) FAILURE DATA

According to the distribution law of the failure data, when the significance level α is 0.1, the confidence interval of the reliability can be obtained as $[0.96, 0.98]$. Let the failure data be denoted as m_1, and the BPA of m_1 can be calculated by formula (6) as follows:

$$\begin{cases} m_1([0.96, 0.98]) = 0.898 \\ m_1([0,1]) = 0.102 \end{cases}$$

(2) EXPERT EVALUATION

According to the general uncertainty theory, the GUV form of GLTs is obtained as: $0.97_{(0.06)}$. Let the expert evaluation be denoted as m_2, and by calculating the BPA of m_2 through formula (6), we have:

$$\begin{cases} m_2([0.94,1]) = 0.936 \\ m_2([0,1]) = 0.064 \end{cases}$$

(3) IMPORTANT PARAMETERS

Similarly, let the important parameters be denoted as m_3, and the BPA is obtained as follows:

$$\begin{cases} m_3([0.92,1]) = 0.913 \\ m_3([0,1]) = 0.087 \end{cases}$$

(4) RELIABILITY TEST

Similarly, let the reliability test be denoted as m_4, and the BPA is obtained as follows:

$$\begin{cases} m_4([0.93, 0.97]) = 0.958 \\ m_4([0,1]) = 0.042 \end{cases}$$

4.2 Obtaining the optimal comprehensive weight

(1) SIMILARITY WEIGHT: Using the information obtained in the previous step, the Jousselme distance is applied to calculate the support degree $Sup(m_i)$ of each piece of evidence and the weight w_i^a. The results are shown in Table 1.

(2) UNCERTAINTY WEIGHT: Using the uncertainty degree u_i° of each piece of evidence, the uncertainty weight of the evidence can be calculated. The results are shown in Table 2.

(3) SUBJECTIVE CONTRIBUTION WEIGHT: Since the subjective weights come from expert evaluations, for the specific calculation formulas, please refer to Formulas (14) and (15). Here, the assigned values of the subjective weights are directly presented, and the results are shown in Table 3.

(4) COMPREHENSIVE WEIGHT: When the similarity weight w_i^a, the uncertainty weight w_i^b, and the subjective weight w_i^c are obtained, the decision matrix A can be obtained as follows:

$$A = \begin{pmatrix} w^a \\ w^b \\ w^c \end{pmatrix} = \begin{pmatrix} 0.249 & 0.249 & 0.247 & 0.255 \\ 0.156 & 0.256 & 0.195 & 0.390 \\ 0.346 & 0.249 & 0.213 & 0.192 \end{pmatrix}$$

According to the TOPSIS method, the optimal comprehensive weight sequence is obtained as follows: $(0.269, 0.246, 0.099, 0.386)$.

Table 1. Similarity Weights of Evidence in the Credal Layer

m_i	$d_{ij(i \neq j)}$	d_{ij}	$Sup(m_i)$	w_i^a
m_1	$d_{12} = 0.033, d_{13} = 0.040, d_{14} = 0.030$	0.103	2.907	0.249
m_2	$d_{23} = 0.037, d_{24} = 0.024$	0.094	2.906	0.249
m_3	$d_{34} = 0.033$	0.110	2.890	0.247
m_4	/	0.087	2.913	0.255

Table 2. Uncertainty Weights of Evidence in the Credal Layer

m_i	u°	w_i^b
m_1	0.10	0.156
m_2	0.06	0.259
m_3	0.08	0.195
m_4	0.04	0.390

Table 3. Subjective Weights

m_i	w_i^c
m_1	0.346
m_2	0.249
m_3	0.213
m_4	0.192

4.3 Modify and fusion of evidence

Modify the original BPA through the optimal comprehensive weight. The results are as follows:

$$m'_1([0.96,0.98]) = 0.242$$
$$m'_1([0,1]) = 0.758$$
$$m'_2([0.94,1]) = 0.230$$
$$m'_2([0,1]) = 0.770$$
$$m'_3([0.92,1]) = 0.090$$
$$m'_3([0,1]) = 0.910$$
$$m'_4([0.93,0.97]) = 0.370$$
$$m'_4([0,1]) = 0.630$$

Please note that the uncertainty information has been integrated into the BPA of different evidence bodies as the interval possibility coverage value at this time.

After completing the revision of the original BPA, the fused evidence is obtained according to Formula (3) of the Dempster combination rule:

$$m([0.92,1]) = 0.033$$
$$m([0.93,0.97]) = 0.215$$
$$m([0.94,0.97]) = 0.065$$
$$m([0.94,1]) = 0.109$$
$$m([0.96,0.97]) = 0.009$$
$$m([0.96,0.98]) = 0.153$$
$$m([0,1]) = 0.335$$

4.4 PPT

There are a relatively large number of pieces of fused evidence, which are not suitable for direct decision-making and judgment. Therefore, it is necessary to perform decision-level probability conversion on multiple pieces of evidence to obtain a result that integrates the core and uncertainty of each piece of evidence. After performing decision-level probability conversion on the fused evidence through Formula (4), the probability distribution of reliability is obtained as follows:

$$BetP(0 \le \hat{r} \le x) = 1 - u^{\circ} = \begin{cases} 0.335x, & 0 \le x < 0.92 \\ 0.745x - 0.3795, & 0.92 \le x < 0.93 \\ 6.1225x - 5.37825, & 0.93 \le x < 0.94 \\ 10.1059x - 9.12265, & 0.94 \le x < 0.96 \\ 26.7559x - 25.10665, & 0.96 \le x < 0.97 \\ 10.2142x - 9.0612, & 0.97 \le x < 0.98 \\ 2.5642x - 1.5642, & 0.98 \le x < 1 \end{cases}$$

Among them, x represents the upper limit of the point estimate of reliability, and u° is the uncertainty degree, which is also the confidence level. This formula reflects how much change in the uncertainty degree (i.e., the confidence level) will be caused by the change in the upper limit value of the point estimate of reliability. It can be concluded from the results that after comprehensively using various types of uncertain reliability information, the confidence level is only 0.579104 when the point estimate value of the reliability of the launch vehicle is $r \ge 0.96$. According to the analysis, since the source process of this reliability assessment result is obtained by synthesizing various types of uncertainty information, there will be a problem that the uncertainty degree further increases during the synthesis process. This is similar to the law that the uncertainty degree of GUVs in the general uncertainty theory further increases during the summation operation, and it also conforms to the human cognitive process.

Based on the results of the operation, it can be considered that it is unreliable to draw the conclusion that the reliability of the launch vehicle is above 0.96 under the confidence level of 90%. Future improvement methods can be carried out from aspects such as increasing the reliability assessment values of the obtained reliability data and reducing the uncertainty of the sources of the reliability data.

4.5 Comparative Analyses

In order to contrastively show the uniqueness of the model proposed in this study, Liu's fusion method of GUVs and the fusion method of D-S evidence theory proposed in this study are selected for comparison. Liu's fusion method is to add different GUVs according to the weights given by experts (Liu & Tang, 2023). Here, the optimal comprehensive weight sequence $(0.269, 0.246, 0.099, 0.386)$, which is obtained in Section 4.2 is selected as the weight for calculation. According to the general uncertainty theory, the simplified forms of the reliability of different types of reliability data are obtained as follows:

For the failure data, based on the failure probability distribution obtained by querying the engineer's manual, the simplified form of the reliability is $\hat{r}_{1(u_1^{\circ})} = 0.97_{(0.1)}$; For expert evaluations, based on the Generalized Linguistic Terms (GLTs) synthesized from the scores given by the expert group, the simplified form of the reliability is $\hat{r}_{2(u_2^{\circ})} = 0.97_{(0.06)}$; For the numerical information of important parameters, the simplified form of the reliability obtained according to the conversion is $\hat{r}_{3(u_3^{\circ})} = 0.96_{(0.08)}$; For the results of the reliability test, the simplified form of the reliability obtained from the rough set reliability interval determined by the test and historical product data is $\hat{r}_{4(u_4^{\circ})} = 0.95_{(0.04)}$. According to the operation rules proposed by (Liu & Tang, 2023), the comprehensive operation results are obtained as follows:

$$\begin{cases} \hat{r} = \sum_{i=1}^{4} w_i \hat{r}_i = 0.97 \times 0.269 + 0.97 \times 0.246 + 0.96 \times 0.099 + 0.97 \times 0.386 = 0.96901 \\ u^{\circ} = \dfrac{1}{\sum_{i=1}^{4} w_i \hat{r}_i} \sum_{i=1}^{4} u_i^{\circ} w_i \hat{r}_i = \dfrac{1}{0.96901} [0.26093 \times 0.1 + 0.23862 \times 0.06 + 0.09504 \times 0.08 + 0.37442 \times 0.04] = 0.063 \end{cases}$$

That is, the final reliability assessment result is $0.96901_{(0.063)}$.

The comparison between the two fusion methods is shown in Table 4. It can be seen that the judgment results obtained by the two theories are not consistent. Among them, the uncertainty degree obtained by using the D-S evidence theory for fusion is greater than the specified value of 0.1, while the operation result obtained by directly adding GUVs meets the operation conditions. For a more intuitive comparison, substituting the estimated value of 0.96901 of $\hat{r}$ obtained by the general uncertainty theory into the PPT formula in Section 4.4, it is obtained that when $\hat{r}$ is less than 0.96901, the value of u° is 0.18. It can be seen that at this time, it is mainly the excessive cognitive uncertainty degree that affects the judgment of the result.

Liu's method of estimating GUVs only considers the influence of subjective weights and the magnitude of the cores of GUVs themselves on the uncertainty degree during the addition process of GUVs. Such a method can be widely applied in the data layer and analysis layer in the reliability assessment pyramid model, because its operation is relatively simple and it can avoid the excessive increase of the uncertainty degree to a certain extent. However, when dealing with the assessment of the decision-making layer at the top of the reliability assessment pyramid model, since it involves the synthesis of reliability data with multi-source uncertainties as a whole, it is difficult to comprehensively reflect the change situation of the uncertainty degree only by using subjective weights and the magnitude of GUVs themselves for the synthesis of the uncertainty degree at this time. To solve this problem, this study uses the evidence theory to fuse GUVs. Meanwhile, the weight distribution among evidences and the comprehensive optimal distribution are considered from three aspects. The obtained results also conform to the results obtained by integrating different uncertain information, and the uncertainty degree will further increase compared with the original information, which conforms to the objective cognitive law. Under the same assessment criteria, the results obtained by the proposed method are more conservative. Moreover, the proposed method can obtain the analytical expression of the change relationship between the assessment result and the uncertainty degree, so it is more conducive to conducting sensitivity analysis. The change relationship between the uncertainty degree and the upper bound of the reliability estimate obtained in this case is shown in Figure 2.

5. Conclusion

The reliability data of complex equipment is characterized by multi-source uncertainties. In order to make better use of reliability data from different sources in practical engineering applications and conduct reasonable reliability prediction and assessment, this study proposes to use the evidence theory to fuse Generalized Uncertainty Variables (GUVs) for reliability assessment. The specific research achievements are as follows:

(1) By linking the concepts of uncertainty degree and confidence level, a method for innovatively converting Generalized Uncertainty Variables in the general uncertainty theory into Basic Probability Assignments in the D-S evidence theory is proposed.

(2) In order to comprehensively consider the similarity among evidences, the uncertainty of evidences themselves, and the coupling relationship among evidences, different ways of weight distribution among different evidences are proposed in these three aspects respectively. Among them, the reciprocal of the uncertainty degree is innovatively applied as the weight distribution for the uncertainty of evidences themselves, and the Shapley value method is applied as the weight distribution for the individual contribution degree of evidences. Finally, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method is used to solve the optimal weight distribution.

Table 4. Comparison of the two GUVs fusion methods

Fusion methods	$\hat{r}$ estimate	u° estimate	Results
D-S evidence theory	≥0.96	0.420896	Reject
General uncertainty Theory	=0.96901	0.063	Accept

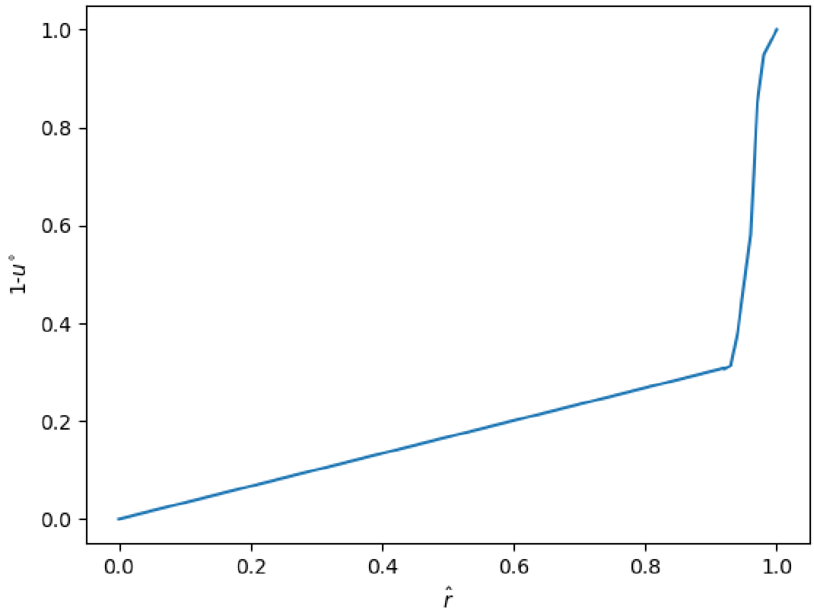

Figure 2. Relationship between the uncertainty degree u° and the reliability $\hat{r}$ in the case.

Compared with the general uncertainty theory, the results obtained by the method proposed in this study are more conservative. Because when evidences containing uncertainty degrees are merged and fused with each other, the uncertainty degree will further increase, which is consistent with the cognitive law in real life. The method proposed in this study will better reflect the impact of differences in confidence levels on the reliability assessment results, so that it can be better applied in reliability engineering practice and help to further improve the reliability management level of complex equipment in China.

Appendix

Abbreviation	Original term
D-S evidence theory	Dempster-Shafer evidence theory
FoD	Frame of Discernment
BPA	Basic Probability Assignment, BPA
TBM	Transferable Belief Model
PPT	Pignistic Probability Transform
GUD	General Uncertainty Data
GUV	General Uncertainty Variable
MTBF	Mean Time Between Failure
MTTR	Mean Time To Repair
UCTs	Uncertain Linguistic Term set
HFLTs	Hesitant Fuzzy Linguistic Term set
EHFLTs	Extend Hesitant Fuzzy Linguistic Term set
GLTs	Grey Linguistic Term set, GLTs
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution

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